COLLEGE of CHARLESTON

MATH MEET 2023

Math Marathon

Instructions

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- Include a cover sheet for each entry with the following information: Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- All entries must be received or postmarked by February 6, 2023.

Problems

1. Find the minimum surface area of a rectangular box with integer sides whose volume is 2023 cubic units.

2. A random number generator generates two numbers a, b uniformly and independently in the interval $[-t, t]$, where $t > 0$. Find the probability, as a function of t, that the graph of the function $y = x^2 + a\,x + b$ intersects the graph of its inverse function.

3. For a positive integer $n \geq 2$, we define a sum of digits function $D(n)$ to be the total number of digits when n is expressed in all of the bases $b = 2, 3, \ldots, n$. For example, since

$$5 = (101)_2 = (12)_3 = (11)_4 = (10)_5$$

we have $D(5) = 9$. Find n such that $D(D(D(n))) = 2023$.

4. In chess, a king may move one square at a time in any direction, either right, left, up, down, or diagonally. How many different ways can a king move from the lower left square to the diagonally opposite upper right square on an 8×8 chess board, using only moves toward the goal (that is, either up, right, or diagonally up-right)?

5. Find positive integers x, y such that $x^2 - 2023\,y^2 = 1$.