Monty Hall All-Day Sprint

The Monty Hall problem has the reputation of being the most confusing probability problem ever posed. Stated in its current form by Steve Selvin in 1975, it entered pop culture in 1990 when Marilyn vos Savant tackled it in her column in *Parade* magazine. Lots of readers wrote in to support her solution, and lots of others wrote in claiming she was wrong. This problem set will help you understand the confusion.

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The name of the problem comes from the whimsical TV game show “Let’s Make a Deal,” hosted by Monty Hall. The show challenged contestants with choices between a small sure prize and something completely unknown: Would you rather have $100 or what’s in this box? The box might contain a valuable prize, but it could also be a “zonk”, something silly like a banana. Near the end of the game, a contestant might be given the opportunity to trade their winnings for a chance at a really big prize hidden behind a door. But behind some of the doors were zonks, so the contestant risked losing everything for a chance at that prize.

The scenario known as the Monty Hall Problem is a simplified final conundrum: There are three closed doors, and the contestant chooses one. Without opening the contestant’s door, Monty opens one of the other two doors, and reveals a goat, so the contestant has clearly avoided a zonk. Monty the asks the contestant: Would you like to keep your original door or would you like to switch and take what’s behind the other closed door? The mathematical question is whether the contestant has better odds of winning the car if they switch to the other door. The exact assumptions used to formulate the problem and interpret the revealed information make a big difference.

We’ll use the following assumptions throughout.

- Behind one door is a car (the prize) and behind the other two are goats (the zonks).
- Initially, the car and goats are equally likely to be behind any door.
- The contestant has no other information about which door hides the car.

The contestant chooses a door, but no doors are opened yet. With this basic information, what is the probability that the car is behind the contestant's first choice of doors?

First scenario. Suppose Monty doesn't know which door the car is behind. Suppose the rules are that after the contestant chooses a door, Monty randomly opens one of the other two doors with equal probability. If the car is revealed, the contestant loses the game at this point. If a goat is revealed, Monty gives the contestant the option to switch doors. Suppose Monty opens a door, and there happens to be a goat behind it. With this additional information, what is the probability that the car is behind the contestant's first choice of doors?

What is the probability that the car is behind the other closed door?

Second scenario. Suppose instead that Monty knows which door the car is behind before the game begins. Suppose the rules are that after the contestant chooses a door, Monty must open a door to reveal a goat. If the contestant picked a door with a goat, Monty must open the other door with a goat. If the contestant picked the door with the car, Monty randomly opens one of the other doors with equal probability, revealing a goat either way. In either case, after he opens the door, what is the probability that the car is behind the contestant's first choice of doors?

What is the probability that the car is behind the other closed door?

FYI: According to Monty himself, the games on *Let's Make a Deal* did not follow either of these idealized scenarios exactly.