Probability All-Day Sprint

CHARLESTON

COLLEGE OF CHARLESTON

MATH MEET

Answer these probability questions. Give exact answers. For some, the answer is just a number, but for others, the answer is in terms of variables in the problem statement. Some of these are really tricky! Write your school or club's answers on the colored answer page and put it in the box near the front door of the Maybank building by 2:00. A winner will be selected at random from the maximally correct submissions.

1. A family has two children. Assume that each child is born a boy or girl with equal probability. Find the conditional probability that they have two girls given at least one of the children is a girl born on the weekend.
2. A total of n bar magnets are placed end-to-end in a line with random independent orientations. Adjacent like poles repel. Ends with opposite polarities join to form blocks. Find the mean number of blocks of joined magnets. Express your answer in terms of n.
3. A stick is broken uniformly at random. Find the mean of the ratio of the length of the shorter piece divided by the length of the longer piece.
4. $U_1, U_2, \ldots, U_n$ are independent random variables each with a uniform distribution on the interval $[0, b]$ where b is an unknown parameter. Find the number R such that $R\sum_i \sqrt{U_i}$ is an unbiased estimate of $\sqrt{b}$. Express your answer in terms of n.
5. Spina bifida is a condition that affects 1 in 1000 babies. The test for this condition is always positive when a baby has spina bifida. When a baby does not have spina bifida, the test comes out negative 95% of the time, and positive 5% of the time. Suppose a doctor tests a baby and it comes out positive. What is the probability that the baby actually has spina bifida?
6. A pack contains m cards labeled $1, 2, \ldots, m$. The cards are dealt out in a random order, one by one. Given that the label of the kth card dealt is the largest of the first k cards dealt, what is the probability that it is also the largest in the pack? Express your answer in terms of m and k.
7. Two points A and B are picked independently and uniformly inside a ball. Find the probability that the sphere with center A and radius AB lies inside the ball.
8. Let $U_1, U_2, \ldots$ be a sequence of independent random variables each uniformly distributed on $[0, 1]$. Define $N = \min\{\, n > 1 \mid U_1 + U_2 + \ldots + U_n > 1 \,\}$. Find the mean of N.
9. Suppose players A and B are playing a game with fair coins. To begin the game, both A and B flip their coins simultaneously. If both A and B get heads (HH), the game ends immediately. If both A and B get tails (TT), they both flip their coins again simultaneously. If one player gets heads and the other gets tails (HT or TH), the player who got heads continues flipping his coin until he get tails. Once they get tails, both players flip their coins simultaneously again. What is the expected number of flips until the game ends?
10. A fair coin is tossed repeatedly until either heads comes up three times in a row or tails comes up three times in a row. What is the probability that the coin will be tossed more than 10 times? Express your answer as a common fraction.