COLLEGE OF CHARLESTON
MATH MEET

# Math Marathon 2025

**Instructions**

- The problems are to be worked out individually and independently. Only textbooks and library sources may be used. Calculators and computers may be used. Each entry must be signed by a math teacher within the school to certify that all rules have been followed. Any number of entries from a school may be submitted.
- Work must be shown neatly and concisely. Explain how you got your answer. It is possible that several entries will have correct solutions, so work will be judged on exposition, clarity of thought and ingenuity, as well as correctness. The date of submission will also be considered. Electronic submissions will be accepted only once.
- Include a cover sheet for each entry with the following information: Student Name, Math Marathon, Home Address, E-mail Address, School; Year of Graduation, School Address, Signature of a Math Teacher for Verification.
- All entrants must be students who have not graduated from high school. All entrants must be registered for the Math Meet.
- The judges' decisions will be final.
- All papers are to be submitted electronically to `mathmeet@cofc.edu` or mailed to the following address

  Math Meet (Marathon)
  Department of Mathematics
  College of Charleston
  66 George Street
  Charleston, SC 29424

- All entries must be received or postmarked by February 14, 2024.

**Problems**

1. Find all expressions of 2025 as a sum of distinct positive integer cubes.

2. A random number generator generates two numbers $a, b$ uniformly and independently in the interval $(0, 1]$. What is the probability that the minimum distance from the graph of $y = a\sqrt{x-b}$ to the graph of its inverse function is greater than $b$ units?

3. An experiment requires a coin to be repeatedly tossed. What is the minimum number of coin tosses required in order to ensure at least 99.99% probability that at least one run of three consecutive heads or three consecutive tails is obtained?

4. For a prime number $p$, let $R_p$ denote the number of distinct remainders modulo 2025 among the powers of $p$; that is, $R_p$ is the cardinality of the set $\{p^n \pmod{2025} : n \in \mathbf{N}\}$. Find the largest and smallest possible values of $R_p$.

5. A random number generator generates two numbers uniformly and independently in the interval $[0, b]$, where $b > 0$. The expected value of the product of their squares is equal to the expected value of the sum of their squares. What is $b$?