College of Charleston
Math Meet 2004
Written Test – Level 1

1. Two women each have two children. The first woman has at least one boy and the second woman's first child is a boy. What are the probabilities for the first woman to have two sons and the second woman to have two sons?

   (A) $\frac{1}{2}, \frac{1}{2}$ (B) $\frac{1}{4}, \frac{1}{4}$ (C) $\frac{1}{3}, \frac{1}{3}$ (D) $\frac{1}{3}, \frac{1}{2}$ (E) $\frac{1}{2}, \frac{1}{3}$

2. What is the distance between the point (x, y) on the line $3x - y + 12 = 0$ and the point $(6, 10)$?

   (A) $x + y - 16$ (B) $\sqrt{10(x^2 + 4)}$

   (C) $\sqrt{(6 - x)^2 + (10 + y)^2}$ (D) $\sqrt{(6 - x)^2 + 4}$

   (E) $\sqrt{10x^2 - 24x + 40}$

3. While traveling from his house to his grandmother's house, George fell asleep when he was half of the distance to her house. When he awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire distance had he been asleep?

   (A) 1/3 (B) 2/3 (C) 1/4 (D) 2/5 (E) 1/2

4. A child collected between 210 and 260 different items on Halloween night. Exactly one seventh of them were cookies and one fourth of them were candies. What is the largest number of items that the child could have collected?

   (A) Between 210 and 219 (B) Between 220 and 229

   (C) Between 230 and 239 (D) Between 240 and 249

   (E) Between 250 and 260

5. The U. S. Department of the Interior has classified some species as follows:
   - T$=\{$x: x is a member of threatened species$\}$
   - E$=\{$x: x is a member of an endangered species$\}$
   - M$=\{$x: x is a mammal$\}$

   Using the symbol $\cup$ to denote the union of sets and the symbol $\cap$ to denote the intersection of sets, express the following in set notation: "The set of all species that are either endangered mammals or threatened mammals."

   (A) $(T \cap M) \cup E$ (B) $(T \cup E)$

   (C) $(T \cap M) \cap E$ (D) $(T \cup E) \cap M$

   (E) none of these

6. The solution to the system of inequalities

$$x + 5y \leq 20$$
$$3x + 2y \leq 21$$

is a region with one corner point (x, y). Give the sum of the coordinates of this corner point.

(A) 0 (B) 8 (C) 9

(D) 41 (E) none of these

7. A quadrilateral with two adjacent sides of lengths a and b is given. Assume $a \geq b$ and a and b are integers. Describe the possible lengths of the diagonal that connects the endpoints of these adjacent sides.

(A) All the integral values strictly between a and b

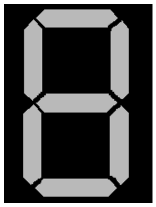
(B) All the real values strictly between a and b

(C) All the real values strictly between $a - b$ and $b + a$

(D) All the integral values strictly between $a - b$ and $b + a$

(E) Question can not be answered with the given information

8. Each letter or digit on the front panel of a microwave oven is comprised of seven individual LED's in the configuration shown. Each LED may be activated (on) or not (off) according to what letter or digit the configuration is to represent. When activated, an LED glows brightly and forms an individual segment of a letter or digit.



A power surge causes the individual LED's to be activated in a random fashion with each segment having a probability of $.5$ of being activated. Find the probability that the configuration of the LED's that are activated causes an even digit to appear on the panel.

(A) $\frac{1}{2^5}$ (B) $\frac{1}{10^7}$ (C) $\frac{5}{10^7}$

(D) $\frac{5}{2^7}$ (E) none of these

9. Find the sum:

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{8 \times 9}$$

(A) $\frac{52}{63}$ (B) $\frac{25}{27}$

(C) $\frac{8}{9}$ (D) $\frac{7}{72}$

(E) None of the above.

10. Given that the five digit number $738t8$ is divisible by 12 and sets $A = \{1,2,3,4,5,6\}$, $B = \{5,6,7,8,9\}$ and $C = \{3,4,5,6,7\}$, which set(s) contain(s) the possible value(s) of the digit t?

(A) Only A (B) Only B (C) A and B (D) B and C (E) C and A

11. In the study of logic, compound statements are represented by expressing simple statements as letters and then symbolically joining the statements, using the following conventions:

- "A and B" is represented by $A \wedge B$
- "A or B" is represented by $A \vee B$
- "not A" is represented by $\sim A$
- "if A then B" is represented by $A \Rightarrow B$

Given the statements:

$$B = \text{My checkbook balances}$$

and

$$C = \text{I have written bad checks,}$$

represent the sentence: If it is true that either my checkbook balances or I have written bad checks and I have not written bad checks, then my checkbook balances.

(A) $(B \vee C) \wedge \sim C \Rightarrow B$ (B) $(B \vee \sim C) \Rightarrow B$

(C) $(B \wedge C) \vee \sim C \Rightarrow B$ (D) $B \Rightarrow (B \vee \sim C)$

(E) none of these

12. A boat is anchored in the middle of a still lake when its fuel tank begins to leak, causing a circular slick on the surface of the lake. If the radius of the slick is growing at a constant rate of 2 ft/min, approximately how much area will the slick cover after 1 hour?

(A) $(120)^2\pi$ square feet (B) $(60)^2\pi$ square feet

(C) $(120)\pi$ square feet (D) $(240)^2\pi$ square feet

(E) 240π square feet

13. A club consists of eight members, including Alonso, Betty, and Carl. How many ways are there to choose a president, secretary, and treasurer if Alonso cannot be president, and either Betty or Carl must be treasurer? (No one may hold more than one office.)

(A) 14 (B) 36 (C) 72 (D) 84 (E) 112

14. A popular fast-food restaurant serves square hamburgers that are $4 \times 4 \times \frac{1}{4}$ cubic inches. What should the radius of a round hamburger that is also $\frac{1}{4}$ inches thick be so that it has approximately the same volume?

(A) $\dfrac{4}{\sqrt{\pi}}$ in. (B) $\dfrac{\sqrt{\pi}}{4}$ in. (C) $\dfrac{16}{\sqrt{\pi}}$ in.

(D) $4\sqrt{\pi}$ in. (E) $\dfrac{8}{\sqrt{\pi}}$ in.

15. Two cars starting from the same point each travel for 3 miles on a straight line going in opposite directions before they turn left and travel 4 more miles each and then stop. What is the distance between them when they stop?

(A) 6 miles (B) 8 miles (C) 10 miles

(D) 14 miles (E) 24 miles

16. In 1980, seventy-five percent of children of the United States were living with two parents. Fifteen years later, that figure changed to sixty-five percent. Assuming that this percentage depends linearly on time, estimate the percentage of children living in two-parent families in 2006.

(A) 26% (B) 58% (C) 62% (D) 68% (E) 72%

17. An interior point of an equilateral triangle is at distances 5, 7 and 8 from the three sides of the triangle. What is the common length of the sides of the triangle?

(A) It cannot be determined

(B) The given configuration cannot exist

(C) 20

(D) $14\sqrt{3}$

(E) $\frac{40}{3}\sqrt{3}$

18. Let l be the tangent line to the circle $x^2 + y^2 = 169$ at the point $(5, -12)$. Find the x-intercept of l.

(A) $\left(\frac{144}{3}, 0\right)$ (B) $\left(\frac{169}{5}, 0\right)$ (C) $\left(\frac{156}{7}, 0\right)$

(D) $\left(\frac{126}{5}, 0\right)$ (E) $\left(\frac{145}{8}, 0\right)$

19. If 70% of the population of North America have seen the Atlantic Ocean and 60% have seen the Pacific Ocean, what is the smallest possible value for the percentage of North Americans who have seen both oceans?

(A) 42% (B) 60% (C) 70% (D) 30% (E) 20%

20. Patrick starts with a cup of cocoa, drinks half of it, then fills the cup up with milk. After stirring and drinking another half-cup of the mixture, he again fills the cup with milk. He continues in this way until he has consumed three cups of the liquid. How much of the original cocoa remains in the cup?

(A) 1/8 (B) 1/16 (C) 1/32 (D) 1/64 (E) 1/128

21. The number of integer solutions for the following simultaneous equations

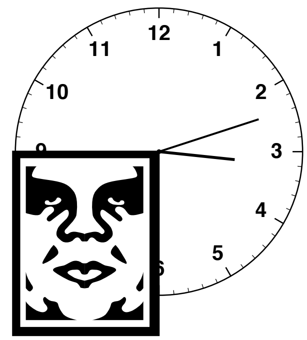
I. $xyz^3 = 24$,
II. $xy^3z = 54$, and
III. $x^3yz = 6$

is:

(A) 1 (B) 2 (C) 3

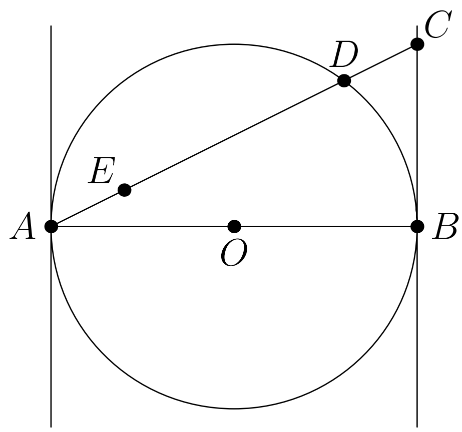
(D) 4 (E) infinitely many

22. A sticker covers one quarter of the face of a clock. For what fraction of the day is it not possible to see both hands?



(A) $\dfrac{3}{16}$ (B) $\dfrac{4}{16}$ (C) $\dfrac{5}{16}$ (D) $\dfrac{7}{16}$ (E) $\dfrac{9}{16}$

23. In the figure, AB is a diameter of the circle with center O and radius r. A chord AD is drawn and extended until it meets the tangent to the circle at B in point C. Then, point E is taken on AC so that $AE = DC$. Let x be the minimum distance from E to the tangent though A and y be the minimum distance from E to the diameter AB. What must be true about x and y?



(A) $y^2 = \dfrac{x^3}{2r - x}$ (B) $y^2 = \dfrac{x^3}{2r + x}$ (C) $y^4 = \dfrac{x^2}{2r - x}$

(D) $x^2 = \dfrac{y^2}{2r - x}$ (E) $x^2 = \dfrac{y^2}{2r + x}$

24. Let S denote the set of all five-digit numbers in which the sum of the digits is equal to 43. Let S' be the subset of S of elements which are divisible by 11. What is the ratio of the size of the set S' to the size of the set S?

(A) $1/3$ (B) $1/5$ (C) $1/6$ (D) $1/11$ (E) $1/15$

25. Find the length of the radius of a circle in which a chord of length 6 is twice as far from the center as a chord of length 12.

(A) $3\sqrt{5}$ (B) $5\sqrt{3}$ (C) $2\sqrt{6}$ (D) $6\sqrt{2}$ (E) $\sqrt{30}$

Answers

1. d
2. b
3. a
4. e
5. d
6. b
7. c
8. d
9. c
10. e
11. a
12. a
13. c
14. a
15. c
16. b
17. e
18. b
19. d
20. d
21. d
22. d
23. a
24. b
25. a