College of Charleston
Math Meet 2013
Written Test – Level 1

1. Which of these choices correctly lists these four numbers in ascending order?

   (A) $2^{500} < 3^{400} < 4^{300} < 5^{200}$ (B) $4^{300} < 3^{400} < 2^{500} < 5^{200}$

   (C) $5^{200} < 2^{500} < 4^{300} < 3^{400}$ (D) $3^{400} < 2^{500} < 4^{300} < 5^{200}$

   (E) $4^{300} < 5^{200} < 3^{400} < 2^{500}$

2. A sample of 500 pounds of sea water was 99% water and 1% salt (by weight). The sea water was boiled until it was 98% water. What is the weight of the remaining solution?

   (A) 490 lbs (B) 470 lbs (C) 450 lbs (D) 350 lbs (E) 250 lbs

3. How many sequences of consecutive positive integers have a sum of 30?

   Note added after Math Meet 2013: Include sequences of length one.

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

4. Let S_1 be the set of points (x, y) in the plane whose coordinates satisfy the equation

   $$(x + y + 2)(x - y + 2) = 0$$

   and let S_2 be the set of points satisfying

   $$(x + y - 2)(y - 2) = 0.$$

   How many points are there in the intersection $S_1 \cap S_2$ (the set of points satisfying both equations simultaneously)?

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. How is the area of a triangle changed if the length of the base of the triangle is *increased* by 10% and the height of the triangle is *decreased* by 10%?

   (A) Decreases by 5% (B) Decreases by 1% (C) Remains the same

   (D) Increases by 2% (E) Increases by 10%

6. What is the sum of the solutions to the equation $2x^2 - 8x + 5 = 0$?

   (A) -8 (B) -5 (C) 0

   (D) 4 (E) None of the above

7. A right triangle in the xy-plane has vertices at the points $(1, 4)$, $(5, -1)$ and $(1, -1)$. The triangle is rotated 90° clockwise around the origin, reflected across the x-axis, translated 4 units right and finally translated 3 units down. What are the coordinates of the midpoint of the hypotenuse of the triangle after these transformations?

   (A) $(3.5, -1)$ (B) $(0, 4.5)$ (C) $(-2, 2.5)$ (D) $(4.5, 1)$ (E) $(5.5, 0)$

8. The *base five* number $0.141414\ldots$ is equal to some base ten fraction. Which fraction is it? (All of the choices are written in base ten)

   (A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{14}{25}$

   (D) $\frac{9}{25}$ (E) none of these

9. If p is the perimeter of a regular hexagon circumscribed about a circle and q is the perimeter of a regular hexagon inscribed in the same circle, then the ratio p/q is:

(A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{3\sqrt{3}}{4}$ (C) $\sqrt{2}$

(D) $\frac{\sqrt{3}}{2}$ (E) none of the above

10. Find the largest two-digit integer that is increased by 75% when its digits are reversed.

(A) 24 (B) 27 (C) 36

(D) 54 (E) None of the above.

11. Let A_n be the average of all whole numbers between 1 and 101 which are multiples of n. (So, for example, A_2 is the average of the numbers $\{2, 4, 6, \ldots, 100\}$.) Which of these is the *largest*?

(A) A_2 (B) A_3 (C) A_4 (D) A_5 (E) A_6

12. At which of these times is the *product* of the numbers before and after the ":" equal to the number of degrees in the angle formed by the hour and minute hand on an analog clock?

(A) 1:05 (B) 2:08 (C) 3:40 (D) 4:42 (E) 5:05

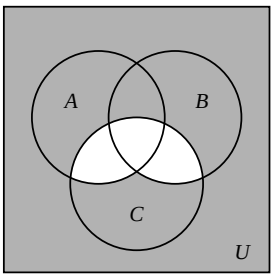
13. Some of the people in a group of x people are divided into y teams of equal size. No one is on more than one team. If there are z people left over, which of the following represents the number of people on each team?

(A) $x - yz$ (B) $x/y + z$ (C) $(x - y)/z$

(D) $(x - z)/y$ (E) none of these

14. If A, B, and C are subsets of the set U, then

- $A \cap B$ is the set of all elements that belong to both A **and** B,
- $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
- A^c is the set of all elements of U that are **not** in A.

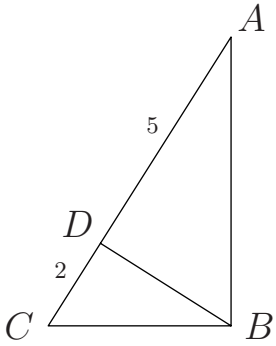


In the diagram, A, B and C are represented by circles and U is represented by a square.
Which of the following expressions describes the region shaded in the diagram?

(A) $(A^c \cap B^c) \cup C^c$ (B) $A^c \cup (B^c \cap C^c)$ (C) $A^c \cap (B^c \cup C^c)$

(D) $(A^c \cup B^c) \cap C^c$ (E) $A^c \cap B^c \cap C^c$

15. In the figure shown, angles ABC and BDC are right angles. The length of segment AD is 5 and the length of segment DC is 2. Find the length of side BC.



(A) 2 (B) $\sqrt{10}$ (C) $\sqrt{14}$ (D) $\sqrt{29}$ (E) $\sqrt{35}$

16. If we know that $\frac{x}{4} - \frac{x}{20}$ is an integer, what must be true about x?

I. x must be a multiple of 4
II. x must be a multiple of 5
III. x must be a rational number which is not an integer

(A) Only I must be true. (B) Only II must be true. (C) Both I and II must be true.
(D) Only III must be true. (E) None of these.

17. What is the probability of getting a sum of sixteen in tossing three standard dice?

(A) $1/6$ (B) $1/27$ (C) $1/36$
(D) $1/12$ (E) None of the above

18. If you completely distribute the multiplication in the expression

$$(a+b+c)^6$$

so that it is a sum of distinct monomials, what is the coefficient of the term a^2bc^3?

(A) 20 (B) $\frac{19}{3}$ (C) 36 (D) $\frac{76}{3}$ (E) 60

19. How many real numbers x satisfy the equation $\frac{1}{x} = \frac{1}{x+1} + \frac{1}{x+2}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

20. In a shop that charges 4% sales tax, always rounding to the nearest cent, which of the following sales totals between $1¢$ and $100¢$ (in whole numbers of cents) is not possible?

(A) 15 (B) 42 (C) 54 (D) 65 (E) 94

21. You have a hat with a white ball in it. You also have a paper bag with one white and one black ball in it. First, you randomly select one of the balls out of the bag (without looking at it) and place it in the hat. Then you randomly select one of the two balls out of the hat. If that ball is white, what is the probability that the ball still in the hat is also white?

(A) 0 (B) $1/3$ (C) $1/2$ (D) $2/3$ (E) 1

22. Given the following four statements:

I. Exactly one of these statements is false.
II. Exactly two of these statements are false.
III. Exactly three of these statements are false.
IV. Exactly four of these statements are false

Assume that each statement is either true or false. Among them, the number of false statements is exactly

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

23. Find the units digit of $1! + 2! + 3! + 4! + \cdots + 1000!$

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

24. Which of the following polynomials has $\sqrt{2} + \sqrt{3}$ as a root?

(A) $(x^2-5)^2 - 24$ (B) $(x^2-2) + (x^2-3)$ (C) $(x^2-2)(x^2-3)$
(D) $x^3 - 2x^2 + 3x - 6$ (E) none of these

25. The longer base of an isosceles trapezoid has a length equal to that of one of its diagonals. The shorter base has length equal to that of the altitude of the trapezoid. If all of these lengths are positive integers, find the smallest possible area of such a trapezoid.

(A) 6 (B) 12 (C) 15

(D) 30 (E) none of these

2013 Answers / Level 1 Test

1. C
2. E
3. E
4. C
5. B
6. D
7. E
8. B
9. E
10. E
11. D
12. B
13. D
14. A
15. C
16. B
17. C
18. E
19. C
20. D
21. D
22. D
23. B
24. A
25. B