College of Charleston
Math Meet 2020
Written Test – Level 1

1. For the polynomial $x^4 + 3x^3 - 5x^2 + 3x + 1$, if a is a root, which of the following must also be a root?

   (A) a^2 (B) $-a^2$ (C) $\dfrac{1}{a}$

            (D) $-\dfrac{1}{a}$ (E) none of these

2. Due to variability in wind speed and direction, when Mary stands at the origin and throws her hat into the air, it will land at one of the 9 points in Figure 1, each with probability 1/9.
   If she throws her hat into the air, walks to where it lands, and throws it into the air again, what's the probability that it will then land at one of the 4 points in Figure 2?

   Figure 1   Figure 2

   (A) 4/81   (B) 9/81   (C) 16/81   (D) 25/81   (E) 36/81

3. Simplify $\sqrt[3]{7 + 5\sqrt{2}} + \sqrt[3]{7 - 5\sqrt{2}}$.

   (A) $2\sqrt[3]{7}$ (B) $\sqrt[3]{14}$ (C) 1

            (D) 2 (E) None of these

4. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles.) What is the area of the largest circle that can be drawn on the floor of this room?

   (A) 16π (B) $\dfrac{81}{4}\pi$ (C) 25π (D) $\dfrac{145}{4}\pi$ (E) 841π

5. A red candle is 1 inch longer than a blue candle. The red candle is lit at 4:30 and the blue candle is lit at 6:00, and at 8:30 they were the same length. The blue candle burned out at 10:00 and the red candle burned out at 10:30.
   Assuming that, as each candle burns, its length decreases at a constant rate, find the original length of the blue candle.

   (A) 2 inches   (B) 5 inches   (C) 8 inches   (D) 11 inches   (E) 14 inches

6. Let S denote the set of all five-digit numbers in which the sum of the digits is equal to 43. Let S' be the subset of S of elements which are divisible by 11. What is the ratio of the size of the set S' to the size of the set S?

   (A) $1/3$ (B) $1/5$ (C) $1/11$ (D) $1/15$ (E) 0

7. Find the sum of all solution(s) x to $|2x - 1| - |x - 5| = 3$.

(A) -4 (B) -1

(C) 1 (D) 3

(E) The equation has no solutions.

8. There are 20 ping pong balls in a bag, each ball is stamped with an integer between 0 and 10 (0 and 10 allowed). The number stamped on each ball, if not 0, is the sum of the numbers stamped on all the other balls. Then the number of balls stamped with 0 is:

(A) no more than 5   (B) 10   (C) 13

(D) 16   (E) at least 18

9. You have three boxes of unknown weight and a scale which only works for items that weigh more than 100 pounds. Since the boxes each weigh less than 100 pounds alone, you decide to weigh them together in pairs. The three different pairs weigh 113, 116 and 117 pounds respectively. By how many pounds does the weight of the heaviest box exceed the weight of the next heaviest box?

(A) 1 pound   (B) 2 pounds

(C) 3 pounds   (D) 4 pounds

(E) cannot tell from information given

10. Let ρ be a rotation about the origin by $51°$ clockwise. What is the minimal number of times one must repeat ρ before every point returns to where it started?

(A) 30   (B) 60   (C) 120

(D) 360   (E) none of these

11. When I write a certain number in base n, it looks like 25. When I write *twice* that number in base n, it looks like 52. Which of these sets *contains* the number n?

(A) $\{6, 11, 20\}$ (B) $\{7, 10, 14\}$ (C) $\{12, 13, 18\}$

(D) $\{8, 15, 19\}$ (E) $\{5, 9, 16\}$

12. Find the distance from the point $(7, 9)$ to the line $x - y = 0$.

(A) 1 (B) $\sqrt{2}$ (C) 2

(D) 10 (E) none of these

13. Evanium is a radioactive substance which decays exponentially with time. In 2 hours, a sample of Evanium will decay to 90% of its original mass. How many hours will it take for a sample to decay to 10% of its original mass?

(A) 18   (B) $\dfrac{2}{1 - \log 9}$ (C) $\dfrac{\ln 10}{\ln 10 - \ln 9}$

(D) $2 \ln \left(\dfrac{1}{9} \right)$ (E) $-\log_{9/10} 10$

14. Six balls numbered 1 through 6 are randomly dropped into six boxes numbered 1 through 6. That is, one ball is dropped in each box, with an equal probability that any given ball ends up in any given box. We say that a ball is in "the right box" if it has the same number as the box that it is in. Which of these events has the *lowest* probability?

(A) no ball ends up in the right box

(B) exactly one ball ends up in the right box

(C) exactly three balls end up in the right boxes

(D) exactly five balls end up in the right boxes

(E) all of the balls end up in the right boxes

15. Find the constant term in the expansion

$$\left(x - \frac{1}{x}\right)^6 = x^6 + \cdots + \frac{1}{x^6}$$

(A) 20 (B) -20 (C) 16 (D) -15 (E) -15

16. Ned says, "A year from now, I will be 4 times as old as my sister was when my brother was born. A year after that, my brother will be 4 times as old as I was when my sister was born. That year, my sister will be 7 times as old as our dog Frieda." All the ages are positive integers. How old is Ned's brother?

(A) Not enough information is available.   (B) 6

(C) 18   (D) 30

(E) Ned is lying.

17. Find the sum of all three-digit positive integers which are multiples of 9.

(A) 54,351   (B) 55,449   (C) 54,540   (D) 55,350   (E) 56,349

18. No Question Selected
19. No Question Selected
20. No Question Selected
21. No Question Selected
22. No Question Selected
23. No Question Selected
24. No Question Selected
25. No Question Selected

2020 Answers / Level 1 Test

1. C
2. D
3. D
4. C
5. C
6. B
7. A
8. E
9. C
10. C
11. D
12. B
13. B
14. D
15. B
16. C
17. D
18. N/A
19. N/A
20. N/A
21. N/A
22. N/A
23. N/A
24. N/A
25. N/A