College of Charleston
Math Meet 2021
Written Test – Level 1

1. C_1 and C_2 are both right circular cones, whose bases are circles of radius 1. Both cones have line segment AB as altitude, but one has its vertex at A, while the other has its vertex at B (so one cone is “upside down" relative to the other). AB has length 1. What is the volume of the intersection of C_1 with C_2?

   (A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$ (E) None of these

2. During his best show, a magician pulled 26 rabbits out of his 5 different hats. We know that
   - he pulled out at least 1 rabbit from the first hat;
   - for each other hat he pulled out more rabbits than from the previous one;
   - he pulled out 4 times as many rabbits from the last hat as from the second hat.

   How many rabbits did the magician pull out of the third hat?

   (A) 3 (B) 4 (C) 5 (D) 6 (E) The given information is contradictory

3. A regular triangle with sides of length 3 is inscribed into a circle. A second circle is inscribed between the triangle and the first circle. Find the radius of the second circle.

   (A) $\frac{\sqrt{3}}{4}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{3}{2}$ (E) None of these

4. Ewa has a flight of 9 stairs to climb. If each step she takes can cover either one or two stairs, how many ways are there for her to reach the 9th stair?

   (A) 38 (B) 45 (C) 46 (D) 55 (E) 64

5. Red played White in a soccer match yesterday. Before the game, there were some predictions:
   - The game will not end in a tie.
   - Red will score against White.
   - Red will win.
   - Red will not lose.
   - Exactly three goals will be scored in the game.

   Exactly three of these turned out to be true. What was the final score?

   (A) Red 2, White 1

   (B) Red 1, White 2

   (C) Red 3, White 0

   (D) Red 0, White 3

   (E) Not enough information was given to determine the score.

6. David tosses a fair coin until it comes up heads and records the number of tosses it takes him. Lacking a coin, Emmy takes a fair six-sided die, labels two sides “heads” and the other four sides “tails,” and then tosses the die until it comes up heads. What is the probability that David’s number of coin tosses equals Emmy’s number of die rolls?

(A) $1/2$ (B) $1/4$ (C) $2/9$ (D) $1/5$ (E) $3/11$

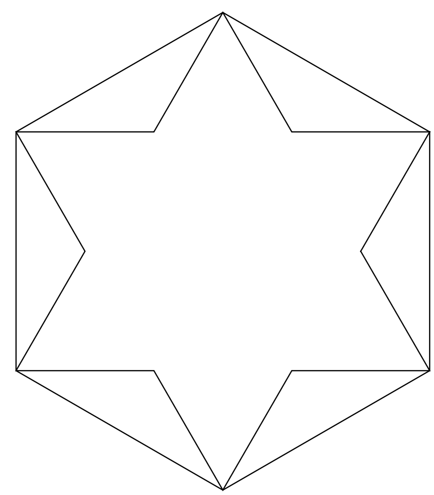
7. A beaker is full of an r% alcohol solution. An identical beaker is 1/3 full of an s% alcohol solution. Two thirds of the full beaker is poured into the 1/3-full beaker and the resulting solution is mixed thoroughly. How many more times must this pouring-and-mixing procedure be repeated so that the difference between the concentrations in the two beakers is less than $0.002|r-s|$?

(A) 2 (B) 3

(C) 5 (D) 8

(E) None of these is the correct answer.

8. Let S be a regular six-pointed star, all of whose acute vertex angles are 60 degrees. Let H be a regular hexagon whose vertices are the six outermost vertices of the star S. What is the ratio of the area of H to the area of S?



(A) 2 to 1 (B) 4 to 3 (C) $\sqrt{3}$ to 1

(D) 3 to 2 (E) None of these

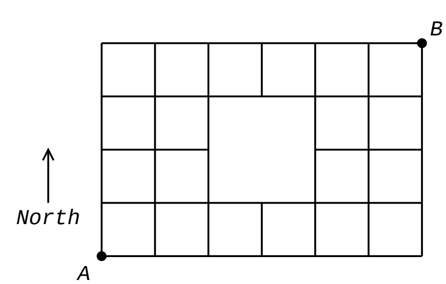
9. Mingus catches some fish. He gives the three largest fish to his dog, thus reducing the total weight of his catch by 35%. He then gives the three smallest fish to his cat, reducing the remaining total weight by 5/13. He ate the remaining fish for dinner. How many fish did Mingus catch?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

10. How many positive integers less than 99 have exactly 2 ones in their binary expansions?

(A) 7 (B) 15 (C) 21 (D) 27 (E) 45

11. An Uber must travel from A to B on the map in the figure. If it never travels south or west, how many different routes can it take?
In the correct answer, the ones digit is . . .



(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

12. Let $ABEF$ be a rectangle. C is a point on edge BE. D is a point on edge AF. $ABCD$ is a square. Suppose that side CE has length 2. If the area of rectangle $ABEF$ is 5, how long is side AB?

(A) $\sqrt{6}-1$ (B) 3 (C) $\dfrac{1+\sqrt{5}}{2}$

(D) $\dfrac{5}{2}$ (E) none of these

13. Two common statistics used to analyze data are the mean and the standard deviation. As you know, the mean is an indication of the "center" of the data. The standard deviation is an indication of the average distance of the data points from the mean.
Professor Jones just analyzed a stack of test grades. He found that the mean of the grades was 82.1 and that the standard deviation was 4.8. Then he noticed that there was one test on the floor that he forgot to include in his computations. The missing test had a grade of 80. How will adding this affect the computed statistics?

(A) it will raise the mean but lower the standard deviation

(B) it will lower the mean but increase the standard deviation

(C) it will increase both the mean and the standard deviation

(D) it will decrease both the mean and the standard deviation

(E) it will decrease the mean but leave the standard deviation the same

14. Suppose A and B are sets of integers such that every element of set A is divisible by 2 and such that at least one element of set B is divisible by 3. Which of these statements must be true?

(A) Every element of $A \cap B$ is divisible by 3.

(B) Some element of $A \cap B$ is divisible by 6.

(C) Every element of $A \cup B$ is divisible by 3.

(D) Some element of $A \cup B$ is divisible by 6.

(E) None of the above.

15. If you toss a fair coin 7 times, what is the probability that no heads ever occur after a tail?

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{16}$ (C) $\dfrac{3}{64}$

(D) $\dfrac{7}{128}$ (E) none of these

16. What are the possible real values for a if

$$\begin{cases} a^3+b^3+c^3 = 3^2 \\ a^3b^3+a^3c^3+b^3c^3 = -3 \\ a^3b^3c^3 = -3^3 \end{cases}$$

(A) $a \in \{0,1,3\}$ (B) $a \in \{\sqrt[6]{3}, -\sqrt[6]{3}, \sqrt[3]{9}\}$

(C) $a \in \{3,-3,0\}$ (D) There are no real solutions

(E) None of these

17. The data below can be modeled by a linear equation.

x	-2	-1	1	2
y	2.1	2.6	3.6	4.1

Find the value of x when $y = 5.4$.

(A) 4.6 (B) 3.37 (C) 0.8

(D) -5.4 (E) none of these

18. Let $p(x)$ be a degree 7 polynomial, all of whose coefficients are integers. If $p(2) = 0$ and $p(3) = 0$, what can we say about $p(5)$?

(A) It's equal to 0

(B) It's divisible by 6

(C) It's greater than 0

(D) It's odd

(E) There isn't enough information to say anything about $p(5)$

19. Find the area of the parallelogram with vertices $(1, 2)$, $(2, 5)$ $(-2, 7)$, and $(-3, 4)$.

(A) 7 (B) 10 (C) 14 (D) 24 (E) 28

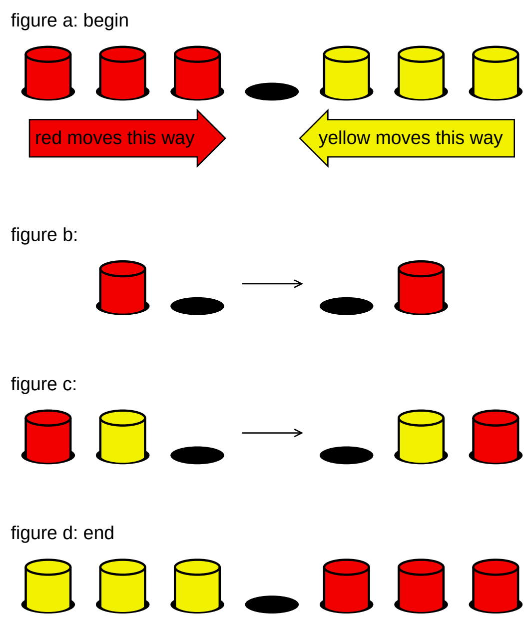
20. For $n \geq 2$, let $a_n = 6a_{n-1} - 9a_{n-2}$ and let $a_0 = \dfrac{1}{2020}$ and $a_1 = \dfrac{1}{2021}$. For $n \geq 1$, define $b_n = \dfrac{a_n}{3^n} - \dfrac{a_{n-1}}{3^{n-1}}$. What can we say about b_n?

(A) $b_n = 3b_{n-1}$ (B) $b_n = -3b_{n-1}$

(C) $b_n = b_{n-1}$ (D) $b_n = 0$

(E) None of the others must be true

21. A game begins with 3 red and 3 yellow pegs placed in 7 holes as shown in figure a.
Pegs may move forward only. The forward direction for red pegs is right; the forward direction for yellow is left.
Two types of moves are permitted: a peg can move one step forward into an empty hole, or jump a peg of the opposing color into an empty hole on the other side (illustrated with red moving in figures b and c). The object of the game is to move pegs, not necessarily alternating between the two colors, until all the yellow pegs are on the left and all the red on the right, as in figure d. In order to achieve this, if red moves first, what colors must move 4th and 5th?



(A) red 4th and 5th.

(B) red 4th, yellow 5th

(C) yellow 4th, red 5th

(D) yellow 4th and 5th.

(E) There is more than one possible answer.

22. A ray of light originates at $(-3, 4)$ in the x-y plane, reflects off of line ℓ at $(0, 0)$ and ends up at $(24, 7)$. What is the equation of ℓ?

(A) $y = -2x$ (B) $y = -x$ (C) $y = -x\sqrt{2}$

(D) $3y = -x$ (E) None of these

23. One side of a six-sided die is labeled 1. An adjacent side is labeled 2. If we start at the 1 and travel from one adjacent side to another, ending up at the 2, visiting each of the six sides exactly once, in how many different orders could we visit the sides of the die?

(A) 6 (B) 8 (C) 9

(D) 12 (E) None of these

24. Starting at the point $(1, 0)$, a bug takes a step of length 1 in the positive y direction, then it rotates 90° to the left and takes a step of length 2, then it rotates 90° left and takes a step of size 3, etc. After the bug has taken 2020 steps, where is it?

(A) $(2020, 2020)$ (B) $(1011, -1010)$ (C) $(505, 505)$

(D) $(2019, -2020)$ (E) None of these

25. "Sonny is 7 years older than Max, but in 5 years, Max will be exactly half Sonny's age." Which of these equations models this problem?

(A) $x + 7 = 2(x + 5)$ (B) $\frac{1}{2}(x + 12) = (x + 5)$ (C) $x + 5 = 2(x + 7)$

(D) $x + 7 = \frac{1}{2}(x - 5)$ (E) $\frac{1}{2}(x + 7) = (x + 5)$

2021 Answers / Level 1 Test

1. A
2. B
3. A
4. D
5. B
6. B
7. C
8. D
9. C
10. C
11. A
12. A
13. D
14. E
15. B
16. B
17. A
18. B
19. C
20. C
21. A
22. D
23. B
24. B
25. B