

**College of Charleston**  
**Math Meet 2022**  
**Written Test – Level 1**

1. Suppose  $p(x)$  is a polynomial of degree 5, and that
  - when  $p(x)$  is divided by  $(x - 2)(x - 3)$ , the remainder is  $4 - x$ ,
  - when  $p(x)$  is divided by  $(x - 1)(x - 3)$ , the remainder is  $x - 2$ , and
  - when  $p(x)$  is divided by  $(x - 1)(x - 2)$ , the remainder is  $3x - 4$ .Find the remainder when  $p(x)$  is divided by  $(x - 1)(x - 2)(x - 3)$ .
  - (A)  $2x^2 - 11x + 16$
  - (B)  $-2x^2 + 9x - 8$
  - (C)  $2x^2 - 7x + 4$
  - (D)  $-2x^2 + 5x - 2$
  - (E) Impossible. There is no such polynomial  $p$ .
2. How many subsets of  $\{1, 2, \dots, 100\}$  are there so that the sum of their elements is 5042?
  - (A) 3
  - (B) 4
  - (C) 6
  - (D) 7
  - (E) 10
3. What is the coefficient of  $x^{8n}$  in  $x^4(2x^2 + 3)^{4n}$ ?
  - (A)  $n6^{3n}$
  - (B)  $(8n)3^{4n-1}$
  - (C)  $216n$
  - (D)  $2^{4n-1}(9n)(4n - 1)$
  - (E) None of these
4. How many fair six-sided dice must you roll in order to have at least a 50% chance of two or more matching dice?
  - (A) 2
  - (B) 3
  - (C) 4
  - (D) 5
  - (E) 6
5. Which of the following is **not** equivalent to the statement “It rhymes when it runs and rains.”?
  - (A) It rhymes or doesn’t run or doesn’t rain.
  - (B) If it rains but not rhymes, then it doesn’t run.
  - (C) When it runs but not rhymes, it doesn’t rain.
  - (D) If it runs, then it rhymes or doesn’t rain.
  - (E) When it rains but not runs, it doesn’t rhyme.
6. Five different numbers are chosen at random from
  - $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ .If any set of five numbers has the same probability of being selected, what’s the probability that their median is 3?
  - (A)  $5/28$
  - (B)  $1/63$
  - (C)  $1/3$
  - (D)  $8/27$
  - (E)  $1/72$
7. Six identical chocolates and five identical cookies are to be distributed (not necessarily evenly) among Amy, Bill, and Carrie. If Amy must receive at least two chocolates and Bill must receive at least one cookie, in how many ways can the chocolates and cookies be distributed? It’s possible that some of the people receive no chocolates at all or no cookies at all.
  - (A) 225
  - (B) 302
  - (C) 360
  - (D) 588
  - (E) None of these

8. Two vertical walls and a horizontal floor meet in the corner of a room. An ant, starting from the corner, walks 1 meter across the floor along a line at a  $45^\circ$  angle to both walls. Another ant, starting from the point 1 meter above the corner, walks 2 meters up the wall along a line at a  $45^\circ$  degree angle from horizontal. Find the distance between the two ants.

(A)  $\sqrt{3 + 4\sqrt{2}}$  (B)  $\sqrt{2 + 4\sqrt{3}}$  (C)  $\sqrt{4 + 2\sqrt{2}}$  (D)  $\sqrt{4 + 3\sqrt{2}}$  (E)  $\sqrt{3 + 3\sqrt{2}}$

9. Let  $x$  be the binary number  $(0.001001001\dots)_2$  and let  $y$  be the octal number  $(0.666666\dots)_8$ . What is  $x + y$  in decimal?

(A)  $0.667667\dots$  (B)  $0.77777\dots$  (C)  $0.838383\dots$   
 (D) 1 (E) None of these

10. The *diameter* of a set is the largest possible distance between two points in the set. If the following shapes all have area 1, which has the largest diameter?

(A) an equilateral triangle (B) a square (C) a regular hexagon  
 (D) a regular octagon (E) a circle

11. I have an original idea for a game, in which the player must guess a passcode consisting of 5 distinct digits (0,1,2,...,9). After each 5-digit guess, the game will give the player some clues as to which digits in her guess appear in the correct answer. See figure. If a player makes the two guesses shown at the bottom of the figure, what is the remainder of the correct answer when it is divided by 9?

**2** A digit on a WHITE background appears in this place in the correct answer.  
**1** A digit on a GRAY background appears in the correct answer, but not in this place.  
**0** A digit on a BLACK background does not appear in the correct answer.

6	9	1	8	7
4	1	0	2	5

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

12. One of the following numbers is not a possible remainder when a perfect square is divided by 167. Which one is it?

(A) 2 (B) 3 (C) 5 (D) 6 (E) 18

13.  $x_n$  is a binary number with  $2n$  bits, half of which are 0 and half of which are 1. As a function of  $n$ , what's the largest possible value for  $x_n$ ?

(A)  $17n^2 - 41n + 26$  (B)  $2^n$  (C)  $(2n - 1)2^n$   
 (D)  $(2^n - 1)2^n$  (E) None of these

14. A biased coin has a probability  $p$  of coming up heads, where  $p$  is a rational number that's less than  $\frac{1}{2}$ . If the probability of two consecutive flips of the coin being identical is  $\frac{145}{288}$ , what's the probability of getting two heads in a row?

(A)  $\frac{5}{24}$  (B)  $\frac{1}{144}$  (C)  $\frac{71}{288}$   
 (D)  $\frac{121}{576}$  (E) None of these

15. How many integers  $x$  are there for which  $x^2 - x + 2$  is divisible by 3?  
 (A) Zero (B) One (C) Three  
 (D) Ten (E) Infinitely many
16. Let  $A$  be the point  $(1, 3)$ . Let  $B$  be the point  $(5, 11)$ .  $\triangle ABC$  is a right triangle with right angle at  $C$ . What is the largest that the area of  $\triangle ABC$  could be?  
 (A)  $10\sqrt{3}$  (B)  $20\sqrt{3}$  (C) 40  
 (D) 80 (E) none of these
17. Joey will leave his house at 4pm and drive at 70mph, arriving at his destination three hours later. However, at 4:23pm, he runs into traffic and has to slow to 50mph for 8 minutes. After that, he returns to driving at 70mph. Then, at 5:47pm he again runs into traffic and has to slow to 48mph for 5 minutes. To make up time so he can still arrive at 7pm, he decides to speed (at a constant velocity) for 30 minutes before returning to 70mph for the rest of the trip. At what velocity does he need to travel during the 30 minutes that he's speeding?  
 (A) 79mph (B) 83mph (C) 90mph  
 (D) 91mph (E) None of these
18. How many real numbers  $0 \leq x \leq 10$  are there for which  $x^2 - 3x + \frac{5}{4}$  is an integer?  
 (A) 10 (B) 11 (C) 71 (D) 75 (E) 100
19. Suppose that we rotate the  $xy$ -plane  $45^\circ$  counterclockwise around the origin and then rotate  $45^\circ$  clockwise about the point  $(1, 1)$ . Which of the following describes the overall motion that has been performed?  
 (A) A translation in the direction of the  $y$ -axis (positive or negative)  
 (B) A translation in some direction other than that of the  $y$ -axis  
 (C) A rotation by  $45^\circ$  (clockwise or counterclockwise) around some point  
 (D) A rotation by some amount other than  $45^\circ$  around some point  
 (E) None of the these
20. If  $x$  is an integer, which of the following must be a divisor of  $x^{320} - x^{170} - x^{151} + x$ ?  
 (A) 4 (B) 13 (C) 17  
 (D) 31 (E) None of the these
21. Let  $C$  be a circle of radius 4 with center  $O$ . Let  $A$  and  $B$  be points on  $C$  so that segment  $AB$  has length 3. Let  $P$  be the point on  $C$  that maximizes the area of triangle  $\triangle ABP$ . How is the area of  $\triangle ABP$  related to the area of  $\triangle ABO$ ?  
 (A) It's double (B) It's 12 more  
 (C) It's triple (D) It's 6 more  
 (E) None of the others must be true
22. A shape in three-dimensional space is first rotated by  $180^\circ$  around the  $y$ -axis and then rotated by  $180^\circ$  around the  $x$ -axis. Which of the following actions will return the shape to its original position?  
 (A) Rotate  $180^\circ$  around  $z$ -axis (B) Translate 1 unit in positive  $x$ -direction  
 (C) Reflect through the plane  $y = z$  (D) Reflect through plane  $x = 0$   
 (E) None of these

