College of Charleston
Math Meet 2023
Written Test – Level 1

1. At my school, there are only three clubs: the math club, the French club, and the history club. The math club has 5 members. The French club has 8 members. The history club has 11 members. Students might be members of more than one club. There are at least 4 people who aren't in the French club. What's the smallest possible number of students at my school?

   (A) 11 (B) 12 (C) 24

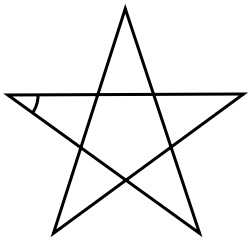
   (D) 28 (E) none of these

2. The graph of $y = x^2 + ax + b$ is symmetric around the line $x = 5$. What's a?

   (A) 5 (B) -5 (C) 10

   (D) -10 (E) none of these

3. How large are the vertex angles in a regular five-pointed star?



   (A) 18° (B) 20° (C) 36°

   (D) 72° (E) none of these

4. Suppose that p and q are relatively prime (meaning no common factors) positive integers for which the solutions to $x^{p/q} = 1$ are $x = \pm 1$. What can we say about p and q?

   (A) p and q are both odd (B) p is even and q is odd

   (C) p is odd and q is even (D) p is odd and q is divisible by 3

   (E) neither p nor q is divisible by 3.

5. Let a and b be positive integers, written in Roman numerals. Which of the following statements are always true?

   1. If the rightmost letter in a is X, then the rightmost letter in the product of a and b is also X.
   2. If the rightmost letter in a is I and the rightmost letter in b is also I, then the rightmost letter in the product of a and b is I.
   3. If the rightmost letter in a is V and the rightmost letter in b is also V, then the rightmost letter in the sum of a and b is X.

   (A) none of them (B) just statement 3 (C) just statements 1 and 3

   (D) just statements 2 and 3 (E) all of them

6. Let x and y be the binary numbers $x = (10000001)_2$ and $y = (10010011)_2$. What is the product $x \cdot y$ in binary?

   (A) $(100001110010011)_2$ (B) $(100100000010011)_2$ (C) $(100100110010011)_2$

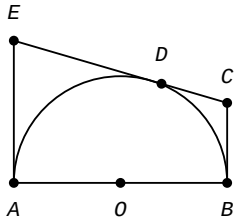
   (D) $(10010010110011)_2$ (E) None of these

7. The figure shows the result of shuffling a deck of 26 cards perfectly one time. How many *additional* perfect shuffles must we perform to return the cards to their original position?

original position:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
after one shuffle:	1	14	2	15	3	16	4	17	5	18	6	19	7	20	8	21	9	22	10	23	11	24	12	25	13	26

(A) 19 (B) 21 (C) 23 (D) 25 (E) 27

8. In the accompanying figure (not to scale), $AOBDA$ is a semicircle centered at O. The lines BC, CE, and EA are tangent to the semicircle at B, D, and A, respectively. If the lengths $\overline{AO} = 1$ and $\overline{AE} = \frac{5}{4}$, what is the area of the trapezoid $ABCE$?
In the correct answer, the second digit after the decimal is

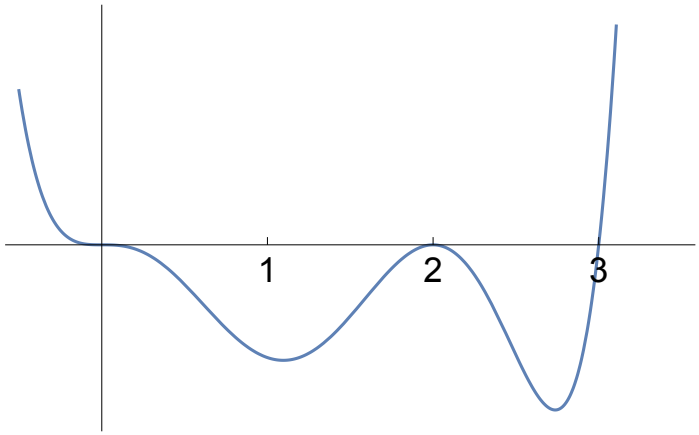


(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

9. A pair of ants are inside a cubical cardboard box with sides 1 meter long. One ant is at the center of the top of the box, and the other is on the bottom, $\frac{1}{4}$ meter from one side and $\frac{1}{8}$ meter from another side.
Assuming both ants stay on the cardboard, how many meters must one crawl to reach the other?
(A) $\frac{1}{8}\sqrt{169}$ (B) $\frac{1}{8}\sqrt{173}$ (C) $\frac{1}{8}\sqrt{187}$ (D) $\frac{1}{8}\sqrt{205}$ (E) $\frac{1}{8}\sqrt{295}$

10. The nation of Torf consists entirely of truth-tellers and liars. Truth-tellers tell the truth 90% of the time, and liars tell lies 60% of the time. But if you ask everyone in Torf whether they're truth-tellers or liars, the number of reported liars will equal the number of actual liars. What part of the population of Torf are truth-tellers?
When the answer is expressed as a fraction between 0 and 1 in lowest terms, the denominator minus the numerator equals
(A) 7 (B) 5 (C) 4 (D) 3 (E) 1

11. The graph shown below is the graph of a polynomial.
Which of the following could be the polynomial?



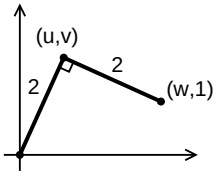
(A) $x(x-2)(x-3)$ (B) $x(x-2)^2(x-3)$ (C) $x^3(x-2)(x-3)$
(D) $x(x-2)^2(x-4)^2$ (E) $x^3(x-2)^2(x-3)$

12. How many solutions are there to the system of equations?

$$xy = 2$$
$$3x + 2y = 15$$

(A) there are no solutions (B) 1 (C) 2
(D) 3 (E) 4

13. Two orthogonal line segments each of length 2 lie in the first quadrant as shown. Find the coordinate v.



(A) $\frac{1}{2}(\sqrt{7}+1)$ (B) $\frac{1}{2}(\sqrt{7}-2)$ (C) $\frac{1}{2}(\sqrt{5}+3)$ (D) $\frac{1}{2}(\sqrt{3}+2)$ (E) $\frac{1}{2}(\sqrt{5}-1)$

14. A logical statement can be thought of as a variable that can take only the values true (T) or false (F). The operators $\wedge$ ("and"), $\vee$ ("or"), $\Rightarrow$ ("implies"), and $\neg$ ("not") act on logical statements to produce other statements. For instance, $p \vee q$ is T when either p or q (or both) is T. $p \Rightarrow q$ is T except when p is T and q is F.
If $(p \wedge \neg q) \Rightarrow (r \vee \neg s)$ is false, how many of the following statements must be true?
I. $(q \wedge r) \Rightarrow (p \wedge s)$
II. $p \wedge (\neg q) \wedge (\neg r) \wedge s$
III. $p \vee q \vee (\neg r) \vee (\neg s)$
IV. $(r \vee \neg s) \Rightarrow (p \wedge \neg q)$ is also false.
V. $p \vee (r \wedge s) \Rightarrow (p \vee r) \wedge (p \vee s)$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. Find the number of real solutions to

$$x^4 - \sqrt{2}x^3 + 2x^2 - 2\sqrt{2}x + 4 = 0$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. What's the smallest $n > 1$ such that $n!$ is divisible by n^4?
(A) 9 (B) 15 (C) 18
(D) 20 (E) None of these

17. The solutions to $x^2 + ax + b = 0$ are prime numbers that are separated by 6. Which of the following is a possible value for b?
(A) 12 (B) 21 (C) 36 (D) 77 (E) 91

18. A **code** consists of a sequence of 10 digits with repetitions allowed, for example, 0034768927. Determine the probability that the sequence **01234** will occur consecutively in a randomly generated code, for example, as it does in 847**01234**62 but does not in 82**01236**4**9**9.
(A) 5.00001×10^{-5} (B) 5.0×10^{-5} (C) 6.0×10^{-5}
(D) 6.0×10^{-10} (E) 5.99999×10^{-5}

19. The Cougars, the College of Charleston Men's Basketball team, has a roster including 6 forwards, 7 guards, and one center. At any time during a game, there will be two guards, two forwards and one center on the floor. How many different line-ups can the Cougars put on the floor during a game assuming that there is no distinction made between the two guard positions and similarly no distinction between the two forward positions?

(A) 42 (B) 315 (C) 2,427

(D) 35,170 (E) more than 1 million

20. The area of the triangle ABC is 24. The centroid of the triangle is P. Line AP meets side BC at the point D. What is the area of triangle BPD?

(A) 4

(B) 6

(C) 8

(D) There isn't enough information to determine the area of BPD

(E) None of these

21. A fair six-sided die has sides labeled: $1,\ 2,\ 2,\ 3,\ 3,\ 3$. If you roll the die twice, what is the probability that the sum of the rolls is 4?

(A) $\frac{1}{3}$ (B) $\frac{2}{7}$ (C) $\frac{4}{15}$

(D) $\frac{7}{36}$ (E) none of these

22. Which of the triples of numbers listed below are the lengths of the edges of a right triangle with perimeter equal to 1000?

(A) $(252, 336, 420)$ (B) $(222, 373, 405)$ (C) $(202, 372, 426)$

(D) $(250, 338, 420)$ (E) $(200, 375, 425)$

23. Let D be the circle $x^2 + 2x + y^2 = 10$. No point on D has integer coordinates (meaning x and y can't both be integers). What is the distance from D to the nearest point with integer coordinates?

(A) $\sqrt{16} - \sqrt{11}$ (B) $\sqrt{11} - \sqrt{8}$ (C) $\sqrt{13} - \sqrt{11}$

(D) $\sqrt{11} - \sqrt{9}$ (E) none of the above.

24. Austin's Famous boxed chocolates come in boxes of 6, 9, and 20 pieces. So, you *can* buy 15 pieces (by buying a box of 6 and a box of 9), but it is not possible to buy exactly 16 pieces. The largest number of pieces that you cannot buy is a two digit number. Give the sum of its digits.

(A) 7 (B) 8 (C) 10

(D) 11 (E) none of these

25. For any two real numbers a and b, define their "diamond product" to be

$$a \diamond b = 4a^2 + 4b^2 - a^2b^2.$$

Which of the following statements are true?

I. The diamond product satisfies the law of commutativity.
II. The diamond product satisfies the distributive law (with respect to ordinary addition of numbers).
III. The diamond product satisfies the associative law.
IV. The equation $c \diamond x = 4c^2$ has one solution for most values of c, but infinitely many for others.
V. There are always exactly two solutions to the equation $c \diamond x = c$ no matter what number c is.

(A) Only I, II and III are true.
(B) Only II and V are true.
(C) Only I and IV are true.
(D) All of the statements are true.
(E) None of the statements are true.

2023 Answers / Level 1 Test

1. B
2. D
3. C
4. B
5. A
6. E
7. A
8. E
9. B
10. E
11. E
12. C
13. A
14. E
15. A
16. E
17. E
18. E
19. B
20. A
21. E
22. E
23. E
24. A
25. C