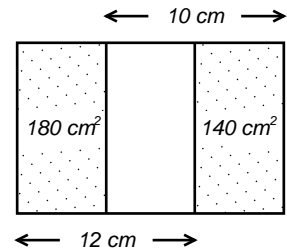


College of Charleston Math Meet 2024 Written Test – Level 1

1. Two horizontal and four vertical lines form three rectangles as shown. (Figure is not to scale.) Find the area in cm^2 of the middle, unshaded rectangle.



- (A) 60 (B) 70 (C) 80 (D) 90 (E) 100
2. Find the number of integer solutions (x, y) to
- $$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$$
- if, for instance, $(x, y) = (2, -4)$ and $(x, y) = (-4, 2)$ are counted as different integer solutions.
- (A) 12 (B) 10 (C) 9 (D) 8 (E) 6
3. Find the remainder when
- $$(t^2 + 9 + (t^3 - 4t)(t^4 + 5t^3 - 4t^2 - 20t))(t - 3 + (2t^2 - 8)(t^3 - 9t^2 + 3t - 27))$$
- is divided by $t^2 - 4$.
- (A) $15t + 20$ (B) $13t - 39$ (C) $-8t - 28$ (D) $-11t + 13$ (E) 111
4. Find the 93rd digit after the decimal point in the decimal form of $\frac{1,234}{99,999}$.
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
5. Let A be the point $(3, 4)$, B be the point $(5, 8)$, C be the point $(-1, -1)$, D be the point $(0, 1)$, E be the point $(1, 3)$, and F be the point $(2, 5)$. Among the triangles $\triangle ABC$, $\triangle ABD$, $\triangle ABE$, and $\triangle ABF$, which has the largest area?
- (A) $\triangle ABC$ (B) $\triangle ABD$ (C) $\triangle ABE$
(D) $\triangle ABF$ (E) They're all the same area
6. \overline{ABCD} is a four-sided polygon. No two edges of \overline{ABCD} cross. Sides \overline{AB} and \overline{CD} are parallel. Find the area of \overline{ABCD} if the lengths of its sides are
- $$AB = 10 \quad BC = 13 \quad CD = 6 \quad DA = 15$$
- (A) 80 (B) 84 (C) 88 (D) 92 (E) 96
7. To win at a certain game, you must toss a coin 5 times and get "heads" each time. If you lose the game, what's the probability that you lost on the first toss? Assume that each time you toss the coin, there's a 50% probability that you'll get "heads."
- (A) $\frac{32}{63}$ (B) $\frac{16}{31}$ (C) $\frac{9}{16}$ (D) $\frac{3}{7}$ (E) $\frac{1}{2}$

8. Which of the following is a divisor of $2^0 + 2^1 + 2^2 + \dots + 2^{34}$?
- (A) 3 (B) 5 (C) 17
(D) 31 (E) None of these

9. If x is a complex number for which $x + \frac{1}{x} = 1$, find $x^5 + \frac{1}{x^5}$.
- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

10. Let a, b, c and d be the positive whole numbers which satisfy

$$\frac{43}{9} = a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}$$

then what is $a \times b \times c \times d$?

- (A) 10 (B) 12 (C) 16 (D) 21 (E) 24
11. Among five digit numbers in which all of the digits are different and none of the digits is 0, what is the probability that the digits are in increasing order?
- (A) $\frac{1}{5}$ (B) $\frac{1}{120}$ (C) $\frac{1}{126}$
(D) $\frac{7}{5000}$ (E) none of these

12. Simplify $(\sqrt{x} - 2)^3 + 6(\sqrt{x} - 2)^2 + 12(\sqrt{x} - 2) + 8$.
- (A) $x^{3/2}$ (B) $x^{3/2} - 4\sqrt{x} + 8$ (C) $x^{3/2} - x - 2\sqrt{x}$
(D) $x^{3/2} - x + \sqrt{x}$ (E) none of these

13. Find the remainder when $(7 + 16871 \times 13)(11 + 45031 \times 13)$ is divided by 13.
- (A) 0 (B) 3 (C) 6 (D) 9 (E) 12

14. Which of the following is equal to $1^{10} \cdot 2^9 \cdot 3^8 \dots 9^2 \cdot 10^1$?
- (A) $1! \cdot 2! \cdot 3! \dots 9! \cdot 10!$ (B) $(10!)^2$ (C) $(10!)!$
(D) $(9!)^9$ (E) none of these

15. It takes Bao 1 hour to walk to school from his home. On Monday, he started walking to school without his homework. His mother realized this and began walking to school with the homework 20 minutes after he left the house. She caught up with him, gave him his homework and then immediately walked home, arriving there 25 minutes after she started.

Assuming they both walked at a constant speed without stopping, how many hours would it take Bao's mother to walk to school from their home?

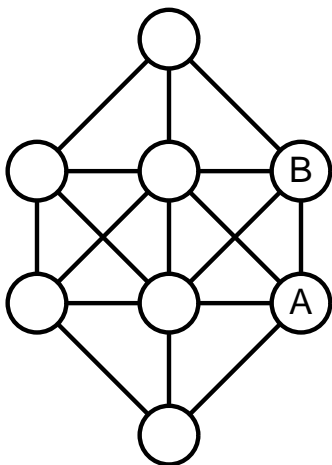
- (A) $\frac{2}{3}$ (B) $\frac{5}{7}$ (C) $\frac{5}{13}$ (D) $\frac{3}{4}$ (E) $\frac{7}{9}$
16. I hold 5 gold coins in my right hand and 7 silver coins in my left. The coins in my right hand weigh 4 ounces more than the coins in my left. After I move one gold coin from my right to my left and one silver coin from my left to my right, the coins in my right hand weigh 12 ounces less than those in my left.
- Assuming each gold coin has the same weight, and each silver coin has the same weight, find the combined weight in ounces of 1 gold and 2 silver coins.
- (A) 62 (B) 60 (C) 59 (D) 45 (E) 42

17. Let p be the polynomial

$$p(x) = (x^2 - 0^2)(x^2 - 1^2)(x^2 - 2^2) \cdots (x^2 - 11^2)$$

It happens to be the case that no matter what integer x is, $p(x)$ is always divisible by 2, by 3, and by lot of other numbers. What's the smallest positive integer n such that there's an integer x for which $p(x)$ *isn't* divisible by n ?

- (A) 13 (B) 16 (C) 23
 (D) 29 (E) none of these
18. Let C be the set of points which are twice as far from the point $(1, 0)$ as they are from the point $(4, 0)$. What is C ?
- (A) a circle (B) a line (C) a parabola
 (D) a hyperbola (E) none of these
19. If x and y are odd integers, which of the following must be an odd integer?
- (A) $x + y$ (B) xy (C) x/y
 (D) $(xy + 1)^2$ (E) none of these
20. An ATM (an automated banking machine) is able to distribute five dollar bills, ten dollar bills and twenty dollar bills. By mistake, an employee puts twenty dollar bills in the five dollar bill slot. As a result, some people get more money out of the machine than they *should* when they make a withdrawal. Which of these amounts could possibly be the amount of money that the bank lost due to this mistake?
- (A) \$9223 (B) \$101 (C) \$99 (D) \$315 (E) \$522
21. Each of the eight circles in the figure is to be filled with exactly one of the digits $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Each digit in this list is to be used exactly once. No two adjacent numbers in this list (2 and 3, for example) may go in circles that are connected by a line. For instance, if A is 2, then B cannot be 1 or 3. (The numbers 1 and 8 are not considered adjacent.) Find B if A is 3.



- (A) 1 (B) 5 (C) 6 (D) 7 (E) 8
22. A bug starts at the origin of the xy -plane. Every second it goes up or right one unit, each with probability 0.5. At one instant, you notice that the bug is at the point $(3,4)$. What is the probability that the bug went through the point $(3,3)$?
- (A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{3}{7}$ (D) $\frac{2}{5}$ (E) $\frac{4}{7}$

23. A pyramid has its peak directly over the center of its base, which is a square with side length 10 cm. If its height is 5 cm, what is the surface area of the pyramid in cm^2 ?
- (A) $25\sqrt{2}$ (B) $50\sqrt{2}$ (C) $100\sqrt{2}$
 (D) $100 + 100\sqrt{2}$ (E) $100 + 200\sqrt{2}$
24. Patrick starts with a cup of cocoa, drinks half of it, then fills the cup up with milk. After stirring and drinking another half-cup of the mixture, he again fills the cup with milk. He continues in this way until he has consumed three cups of the liquid. How much of the original cocoa remains in the cup?
- (A) $1/8$ (B) $1/16$ (C) $1/32$ (D) $1/64$ (E) $1/128$
25. $\frac{10}{1} + \frac{9}{2} + \frac{8}{4} + \frac{7}{8} + \cdots + \frac{1}{512}$ is very close to being 18. How far is it from 18?
- (A) $\frac{1}{1024}$ (B) $\frac{1}{512}$ (C) $\frac{3}{1024}$
 (D) $\frac{3}{512}$ (E) none of these

2024 Answers / Level 1 Test

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|------|-------|-------|
| 1. A | 10. E | 19. B |
| 2. C | 11. B | 20. D |
| 3. B | 12. A | 21. B |
| 4. C | 13. E | 22. E |
| 5. E | 14. A | 23. D |
| 6. E | 15. C | 24. D |
| 7. B | 16. A | 25. B |
| 8. D | 17. D | |
| 9. D | 18. A | |