College of Charleston
Math Meet 2014
Written Test – Level 2

1. A mixture of viruses and bacterial cells are placed in a test tube. As time passes, the viruses *bind* to the cells so that the number of bound viruses at time t minutes is approximated by the function

$$N(t) = \left(1 - e^{-kt}\right) N_0$$

where N_0 is the total number of viruses in the mixture. According to this model, after how many minutes are 50% of the viruses bound?

(A) $\ln(k^2)$ (B) $\ln(2)/k$ (C) $\ln(k/2)$ (D) $k^{-1}\ln(1/2)$ (E) $-\ln(k^2)$

2. Spinner A has a probability of 0.6 of landing on green and 0.4 of landing on red. Spinner B has probability 0.2 of landing on green and 0.8 of landing on red. What is the probability that they land on the same color?

(A) 0.44 (B) 0.50 (C) 0.56 (D) 0.60 (E) 0.64

3. Let x be the solution to $\sqrt{x - \sqrt{26 + x}} + \sqrt{x + \sqrt{26 + x}} = 6$. Which of these statements is true?

(A) $x - 1$ is a prime number (B) $x - 2$ is a prime number (C) $x - 3$ is a prime number (D) $x > 14$ (E) $x^2 < 87$

4. Two four-digit numbers $abcd$ exist such that

$$abcd + a + b + c = 2014.$$

What is the product of the digits in the smaller of the two?

(A) 0 (B) 2 (C) 288 (D) 324 (E) 405

5. Find the number of points of intersection between the two curves $1 - 6xy + 3y^2 = 0$ and $1 + 6xy + 3x^2 = 0$.

(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

6. Suppose that $p(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers. Find d if $p(1 - i) = p(-1 + i\sqrt{2}) = 0$.

(A) -2 (B) 0 (C) 2 (D) 4 (E) 6

7. Three cards are placed into a hat. One card is red on both sides. One card is blue on both sides. But, the third card is red on one side and blue on the other. If one of the cards is selected randomly from the hat and the side you can see is red, what is the probability that it is also red on the side you have not seen?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

8. Given below are three sets of points.

I. $\{(-1,0),(-2,1),(-1,3)\}$
II. $\{(3,-6),(3,-3),(4,-3)\}$
III. $\{(1,1),(4,3),(-3,7)\}$

Which, if any of these sets, represent the vertices of a right triangle?

(A) I only (B) II only (C) II and III only

(D) I and II and III (E) none of these

9. If $f(x) = \ln(x^2)$ and $g(x) = 2\ln(x)$, then which of the following is true?

(A) $f(x) = g(x)$ for all real numbers x.

(B) $f(x) = g(x)$ for all real numbers in the domain of g.

(C) $f(x) = g(x)$ for all real numbers in the domain of f.

(D) $f(x) = g(x)$ except for $x = 0$.

(E) $f(x) \neq g(x)$ for all real numbers x.

10. Which of the following trigonometric identities is true for all values of θ such that $0 < \theta < \pi/2$?

(A) $\tan^2(\theta)\sin^2(\theta) = \tan^2(\theta) - \sin^2(\theta)$

(B) $\tan^2(\theta) + 1 = \csc^2(\theta)$

(C) $\tan^2(\theta)\cos(\theta) = \cot^2(\theta)\sin(\theta)$

(D) $\tan(\theta)\sin(\theta) = \tan(\theta) + \cos(\theta)$

(E) $\tan(\theta) + \sin(\theta) = \cos(\theta) - \tan(\theta)$

11. The course grade for students in Ms. Noether's math class is based on the sum of the three 100 point tests they take. One student had to miss a test for a good reason. Ms. Noether is considering two schemes to correct this problem in a fair way. Under Scheme A, she will pretend that the student's score on the missing test was the average of the other two scores. Under Scheme B, she will multiply the sum of the two scores from tests taken by $300/200$ to "scale it up" so that it is a score out of 300 points. Under which scheme will the student have a higher grade?

(A) Scheme A always gives a higher grade.

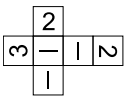
(B) Scheme B always gives a higher grade.

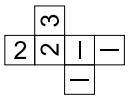
(C) Scheme A if the average of the two tests taken is above $66.\overline{66}$.

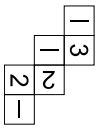
(D) Scheme A if the average of the two tests taken is below $66.\overline{66}$.

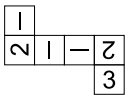
(E) The two schemes always give the same value.

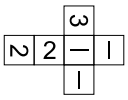
12. The shape in the figure to the right can be folded up to form a cube. Which of the following shapes can be folded up to form a cube that is congruent to it?

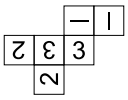


(A) 

(B) 

(C) 

(D) 

(E) 

13. There is a famous way to encode a word as a number by using powers of primes. Thinking of each letter as a number indicating its position in the alphabet, you encode the word by raising the first prime to the power of the first letter in the word, the second prime to the power of the second letter in the word and so on until you get to the end of the word. For example, since "D" is the 4th letter, "O" is the 15th letter and "G" is the 7th letter, the letter sequence "DOG" would be encoded as

$$2^4 \times 3^{15} \times 5^7 = 17,936,133,750,000.$$

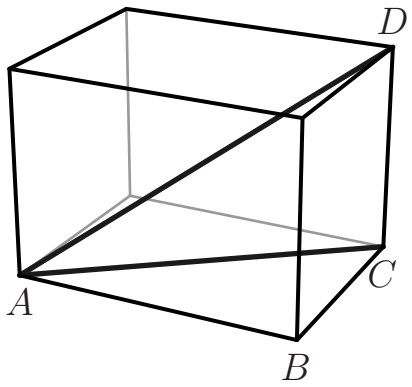
Using this system, what three letter sequence is represented by the number 600?

(A) ABC (B) BAC (C) BCA (D) CBA (E) CAB

14. A group of good friends go out to dinner at a restaurant and agree that they will split the cost evenly. However, by the time the bill totaling $\$87.50$ has arrived, two of them have left without paying. (Apparently, they were not really such good "friends" after all.) The remaining dinner guests are a little angry, but realize they only have to pay an extra $\$5$ each to cover the cost. How many people were in the original group?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

15. The figure shows a rectangular solid with length $AB = 4$, width $BC = 3$, and height $CD = 5\sqrt{3}/3$. Find the angle CAD.



(A) $\dfrac{\pi}{12}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{4}$

(D) $\dfrac{\pi}{3}$ (E) None of the above.

16. The output of the function

$$f(x) = \sqrt{6 - \sqrt{2x + 8}}$$

is a real number precisely when the real variable x satisfies $a \leq x \leq b$. What is $a + b$?

(A) 10 (B) 12 (C) 14 (D) 16 (E) 18

17. A student who believed that common digits could be cancelled from the numerator and denominator of a fraction would think that

$$\frac{23}{35} = \frac{2}{5}.$$

Of course, this is *not* correct. For which of the following fractions would that misguided logic, just by coincidence, produce the correct result?

(A) $\dfrac{24}{45}$ (B) $\dfrac{25}{55}$ (C) $\dfrac{26}{65}$ (D) $\dfrac{27}{75}$ (E) $\dfrac{28}{85}$

18. A signal is made by placing 3 flags, one above the other, on a flagpole. If there are 7 flags available and each one looks different than the others, how many signals are possible?

(A) 21 (B) 210 (C) 35 (D) 350 (E) 735

19. The light rays from a spotlight spread out to form a solid cone, with the light rays making an angle of up to 30 degrees with the centerline of the beam. If the spotlight is 10 feet above the floor, and the centerline of the beam hits the floor at a 60-degree angle of inclination, what is the area the spotlight covers on the floor?

(A) 25π (B) 75π (C) $100\pi/\sqrt{3}$ (D) $400\pi/9$ (E) $25\sqrt{6}\pi$

20. What is the sum of the solutions to $\ln(2e^{2x} - 8e^x + 10) = x$?

(A) 4 (B) 5 (C) $\ln 4$ (D) $\ln 5$ (E) none of these

21. What is the *product* of the two solutions to $f\left(f(x)\right) = x^2$ where $f(x) = -3x + 2$?

(A) -4 (B) 4 (C) -9 (D) 9 (E) 12

22. Let $p(x)$ be a polynomial of the form $p(x) = ax^2 + bx + a$ which has $(3x - 2)$ as a factor. What is $\dfrac{p(0)}{p(1)}$?

(A) -6 (B) -5 (C) 0 (D) 5 (E) 6

23. Let S denote the set of all five-digit numbers in which the sum of the digits is equal to 43. Let S' be the subset of S of elements which are divisible by 11. What is the ratio of the size of the set S' to the size of the set S?

(A) $1/3$ (B) $1/5$ (C) $1/11$ (D) $1/15$ (E) 0

24. On a clock, the hands move continuously so that the minute hand makes a complete revolution every hour and the hour hand makes a complete revolution every twelve hours. Exactly how many hours are there between consecutive times that the hands happen to form a right angle?

(A) It is not always the same amount of time between consecutive right angle configurations.

(B) Always $\dfrac{3}{8}$ of an hour.

(C) Always $\dfrac{4}{9}$ of an hour.

(D) Always $\dfrac{5}{10}$ of an hour.

(E) Always $\dfrac{6}{11}$ of an hour.

25. What is the remainder when the polynomial $202x^2 + 2014$ is divided by $x - 10$?

(A) -22014 (B) 20214 (C) 22101 (D) 22214 (E) 241022

2014 Answers / Level 2 Test

1. B
2. A
3. C
4. E
5. E
6. E
7. D
8. C
9. B
10. A
11. E
12. C
13. E
14. A
15. B
16. A
17. C
18. B
19. E
20. D
21. B

22. A
23. B
24. E
25. D