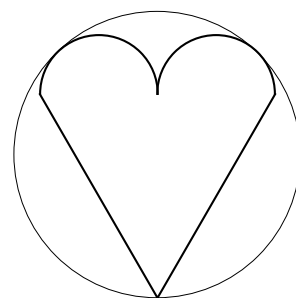


College of Charleston
Math Meet 2015
Written Test – Level 2

1. Take an equilateral triangle of side length 2. Pick one side of the triangle and attach two semicircles of diameter 1. What is the radius of the smallest circle that can contain this heart-shaped figure?



- (A) $\frac{3}{2\sqrt{3}-1}$ (B) 1 (C) $\frac{2}{\sqrt{3}}$
 (D) $\sqrt{3}$ (E) none of these
2. L and K are perpendicular lines in the xy -plane. The three lines L , K , and $y = 4 - x$ intersect at a single point. L 's y -intercept is 7, and K 's x -intercept is 1. Find the sum of the slopes of L and K .
- (A) 0 (B) $2/3$ (C) $-3/2$ (D) $-2/3$ (E) $3/2$
3. A coin is biased so that the probabilities of heads and tails are $2/3$ and $1/3$ respectively. A second coin is biased so that the probabilities of heads and tails are $3/5$ and $2/5$ respectively. If both coins are tossed, find the probability of at least one head.
- (A) $\frac{13}{15}$ (B) $\frac{2}{5}$ (C) $\frac{8}{15}$
 (D) $\frac{3}{5}$ (E) None of the above
4. Given $f(x) = f(x - 2) + 3$ for all x , what is the distance between the points $(1, f(1))$ and $(3, f(3))$?
- (A) $\sqrt{3}$ (B) 5 (C) 7 (D) $\frac{17}{4}$ (E) $\sqrt{13}$
5. Professor Calini always tells the truth, except on the weekend (Saturday and Sunday), when she lies all the time. On how many days of the week can Professor Calini say "If I did not tell the truth yesterday, then I will not lie tomorrow?"
- (A) 0 (B) 1 (C) 3 (D) 5 (E) 7
6. Solve the inequality $4 - 8(x - 2) > 2(x + 1) - 7x$.
- (A) $x < 6$ (B) $x > -6$ (C) $x < -18$
 (D) $x > 6$ (E) none of these

7. Which of the given functions satisfies

$$f(n+1) - f(n) - 6f(n-1) = 0$$

for all integers n ?

- (A) $f(n) = 3 \cdot 2^n - 2 \cdot 3^n$ (B) $f(n) = 3 \cdot 2^n + 2 \cdot (-3)^n$
(C) $f(n) = 1 - 2 \cdot 3^n$ (D) $f(n) = 3 \cdot (-2)^n + 2 \cdot (-3)^n$
(E) $f(n) = 3 \cdot (-2)^n - 2 \cdot 3^n$

8. Simplify $\sin(x) + \cos(x) + \sec(x) + \csc(x)$.

- (A) 1
(B) $(\csc(x) + \sec(x))(1 + \sin(x) \cos(x))$
(C) $\cot(x) + \tan(x)$
(D) $\frac{\csc(x) \sec(x)}{\cot(x) + \tan(x)}$
(E) none of these

9. Let C be the circle $x^2 + y^2 = 27$. No point on C has integer coordinates (meaning x and y can't both be integers). What is the distance from C to the nearest point with integer coordinates?

- (A) $\frac{1}{\sqrt{27}}$ (B) $\frac{1}{3}$ (C) $\sqrt{3} - \sqrt{2}$
(D) $\sqrt{27} - \sqrt{26}$ (E) none of these

10. Let x be a real number between .9 and 1. List the numbers x , y and z in order of increasing magnitude where

$$y = x^x \quad \text{and} \quad z = x^y.$$

- (A) $x < y < z$ (B) $x < z < y$ (C) $z < x < y$ (D) $z < y < x$ (E) $y < x < z$
11. How many different numbers can be written in the form $x + y + z$ where x , y , and z are in $\{1, 3, 5, \dots, 21\}$?
(A) 9261 (B) 4830 (C) 441 (D) 231 (E) 31
12. A professor distributes 20 sample mathematics problems and says that the test will consist of 10 problems chosen from these 20. What is the minimum number of these sample problems that a student must know how to do in order to guarantee that they will pass the test with at least a grade of 60%?
(A) twenty questions (B) eighteen questions (C) sixteen questions
(D) fourteen questions (E) none of the above

13. If you roll an ordinary six-sided die three times, what is the probability that you will get three consecutive integers in increasing order?

- (A) $\frac{1}{216}$ (B) $\frac{1}{108}$ (C) $\frac{1}{72}$ (D) $\frac{1}{54}$ (E) $\frac{5}{216}$

14. Which of the following numbers is a solution to $15 \sin x + 12 \tan x = 28$?

- (A) $\tan^{-1} \frac{4}{3}$ (B) $\sin^{-1} \frac{15}{17}$ (C) $\tan^{-1} \frac{12}{5}$ (D) $\tan^{-1} \frac{24}{7}$ (E) $\sin^{-1} \frac{3}{7}$

15. A speaker lectured to an audience for an hour. During that hour, ten percent of the audience slept through the entire lecture and twenty percent heard the entire lecture. Half of the remainder of the audience heard one-third of the lecture, and the other half heard two-thirds of the lecture. What was the average number of minutes of the lecture heard by the members of the audience?
- (A) 24 (B) 30 (C) 33
(D) 36 (E) none of these
16. Suppose that the College of Charleston always holds the Math Meet on the second Saturday of February. This year, that happens to be February 14th (Valentine's Day). Which year would be the next time that the Math Meet is on Valentine's Day?
- (A) 2020 (B) 2022 (C) 2026
(D) 2043 (E) none of these
17. Let $ABCDEFGHIJKL$ be a regular 12-gon. Consider the following angles: $\angle CAB$, $\angle DAC$, $\angle EAD$, $\angle FAE$. Which one of the following statements is true about those four angles?
- (A) $\angle CAB$ is larger than the rest (B) $\angle DAC$ is larger than the rest
(C) $\angle EAD$ is larger than the rest (D) $\angle FAE$ is larger than the rest
(E) all four angles all the same size
18. $(\log_2 4)(\log_3 5)(\log_4 6)(\log_5 7)(\log_6 8)(\log_7 9) =$
- (A) 6 (B) 5 (C) 4
(D) 3 (E) none of these
19. Which of the following expressions is equal to $\frac{1}{x} - \frac{1}{x^3} + \frac{1}{x^5} - \frac{1}{x^7} + \frac{1}{x^9} - \frac{1}{x^{11}}$?
- (A) $\frac{1}{x^{36}}$ (B) $\frac{x^{12} - 1}{x^{13} + x^{11}}$ (C) $\frac{x^{12} - x}{x^{11}}$
(D) $\frac{x^{10} - 1}{x - 1}$ (E) none of these
20. A set of test grades is averaged. Later, one more grade is added to the set. The new test grade is p points above the previous average and as a result the average increases by q points. How many test grades were in the original set?
- (A) $\frac{p}{q} - 1$
(B) $\frac{q}{p}$
(C) $\frac{p + q}{q}$
(D) $\frac{p - q}{p}$
(E) not enough information provided to know
21. How many values of θ in $[0, \pi]$ satisfy $\cos 2\theta \cos \theta = 1 + \sin \theta \sin 2\theta$?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 5

22. $ABCD$ is a square, and E is a point in the interior of the square. If $\overline{AE} = 1$, $\overline{BE} = 2$, and $\overline{CE} = 3$, find \overline{DE} .
- (A) $\sqrt{4}$ (B) $\sqrt{6}$ (C) $\sqrt{10}$ (D) $\sqrt{12}$ (E) $\sqrt{13}$
23. A parade is two miles long and moves with constant speed. A person who is at the end starts walking towards the front of the parade, moves forward with a constant speed until he reaches the front, and then turns around and walks at the same speed until he reaches the end of the parade. By this time the parade has moved two miles forward. What distance has the person walked?
- (A) 4 (B) 2 (C) $2(1 + \sqrt{3})$ (D) $2(1 + \sqrt{2})$ (E) $3(1 + \sqrt{2})$
24. $(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})^4 =$
- (A) $\frac{1}{2} + i \frac{\sqrt{3}}{2}$ (B) $\frac{1}{2} - i \frac{\sqrt{3}}{2}$ (C) $-\frac{1}{2} + i \frac{\sqrt{3}}{2}$ (D) $-\frac{1}{2} - i \frac{\sqrt{3}}{2}$ (E) -1
25. If $2 \log(x - 2y) = \log x + \log y$, find x/y . Assume that x , y , and $x - 2y$ are strictly greater than zero.
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2015 Answers / Level 2 Test

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| 1. A | 10. B | 19. B |
| 2. C | 11. E | 20. A |
| 3. A | 12. C | 21. C |
| 4. E | 13. D | 22. B |
| 5. D | 14. A | 23. D |
| 6. A | 15. C | 24. C |
| 7. E | 16. C | 25. D |
| 8. B | 17. E | |
| 9. D | 18. A | |