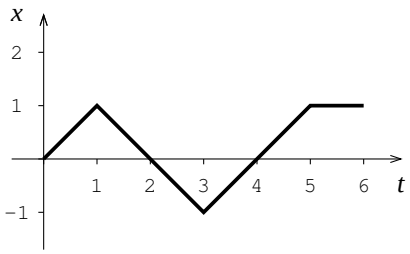
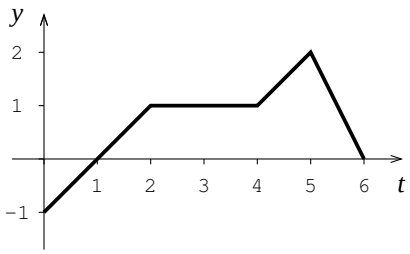
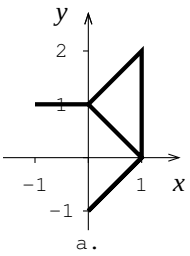
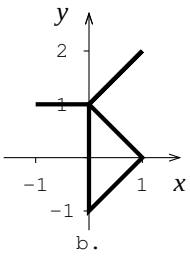
College of Charleston
Math Meet 2016
Written Test – Level 2

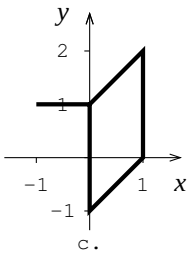
1. An insect crawled about on the x-y plane for six seconds, and as it did, I plotted its x and y coordinates as functions of time t. Which x-y graph shows the path of the insect?

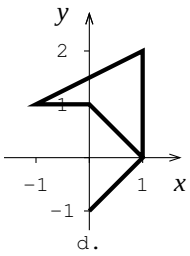


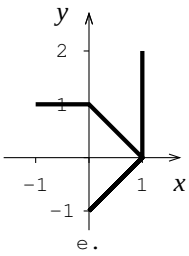












(A) figure a. (B) figure b. (C) figure c. (D) figure d. (E) figure e.

2. Which of the choices is equal to $\dfrac{x}{1+\frac{x^2-2x-1}{x+1}}$ for every value of x other than 0, 1 and -1?

(A) $\dfrac{x-1}{x+1}$ (B) $\dfrac{x+1}{x-1}$ (C) $\dfrac{x}{x+1}$ (D) $\dfrac{x}{x-1}$ (E) $\dfrac{x^2+x}{x-1}$

3. How many different numbers can be written in the form $x+y$ where x and y are in $\{1, 2, 4, \ldots, 2^{11}\}$?

(A) 21 (B) 32 (C) 66 (D) 78 (E) 121

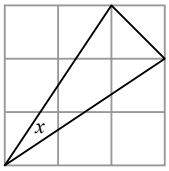
4. If $p(x)$ is a polynomial, and if $p(x)+ip(1-x)=\sqrt{2}x^5$, what's the coefficient of x^5 when $p(x)$ is written in the standard power form $p(x)=a+bx+cx^2+\cdots$? (Here, i denotes $\sqrt{-1}$.)

(A) $1-i$ (B) $\sqrt{2}(1-i)$ (C) $\sqrt{2}(1+i)$ (D) $\dfrac{1-i}{\sqrt{2}}$ (E) $\dfrac{1+i}{\sqrt{2}}$

5. If $f(x)=\dfrac{2x+3}{5x-7}$, then $f^{-1}(x)=$

(A) $\dfrac{3x+5}{7x-2}$ (B) $\dfrac{2x-7}{5x+3}$ (C) $\dfrac{5x+2}{3x-7}$ (D) $\dfrac{7x+3}{5x-2}$ (E) $\dfrac{5x-7}{2x+3}$

6. Let x be the angle at (0,0) in the triangle with vertices (0,0), (3,2), and (2,3). Find $\sin x$.



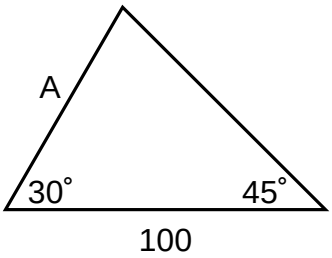
(A) 3/13 (B) 7/26 (C) 4/13 (D) 9/26 (E) 5/13

7. $\lfloor x \rfloor$ denotes the greatest integer which is less than or equal to x. For example, $\lfloor 6.25 \rfloor = 6$, $\lfloor 4 \rfloor = 4$, and $\lfloor \pi \rfloor = 3$. What can you say about the integer solutions n of the following equation?

$$\lfloor \sqrt[4]{1} \rfloor + \lfloor \sqrt[4]{2} \rfloor + \lfloor \sqrt[4]{3} \rfloor + \cdots + \lfloor \sqrt[4]{n} \rfloor = 2n$$

(A) There are no solutions.
(B) There is more than one solution.
(C) The only solution is $n = 85$
(D) The only solution is $n = 90$
(E) The only solution is $n = 95$

8. Find the length of side A in the triangle.



(A) $200/\sqrt{3}$ (B) $100(\sqrt{3}-1)$ (C) $100\sqrt{3}$
(D) 50 (E) $100/\sqrt{2}$

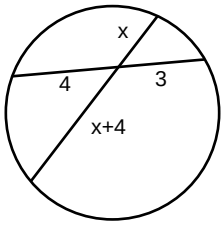
9. A jar is filled with two types of candies: lemon drops and mint drops. Each piece of candy is wrapped either in red paper or white paper.
7/10 of the candy is in red paper.
3/10 of the candy consist of mint drops wrapped in red paper.
3/5 of the candy is lemon drops.
If you pull a mint drop from the jar, what's the probability that it is wrapped in white paper?
(A) 1 (B) 4/5 (C) 1/2 (D) 2/5 (E) 1/4

10. Let $\lfloor x \rfloor$ denote the greatest integer n such that $n \leq x$ and define the function

$$f(x) = \left(\lfloor x \rfloor + \lfloor -x \rfloor\right)^5.$$

Which of these best describes the *range* of the function f?
(A) The range of f is the set of all real numbers.
(B) The range of f is an open interval.
(C) The range of f is a closed interval.
(D) The range of f is a set containing exactly one element.
(E) The range of f is a set containing exactly two elements.

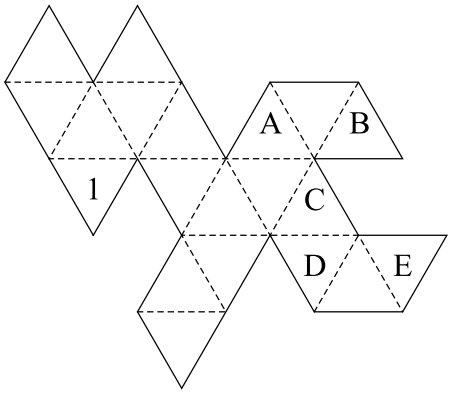
11. A *chord* is a line segment whose endpoints lie on a circle.
Two chords lie in the same circle. If the first divides the second into lengths 3 and 4, and the second divides the first into lengths x and x+4, find x.



(drawing not to scale)

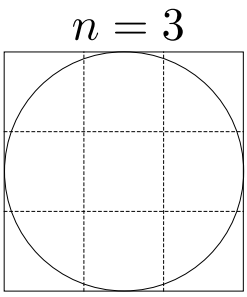
(A) 2 (B) $-1+\sqrt{13}$ (C) $2+\sqrt{5}$ (D) $3-\sqrt{5}$ (E) $5-\sqrt{2}$

12. The figure to the right could be folded up to form an icosahedron. Which face would be opposite the face labeled 1?



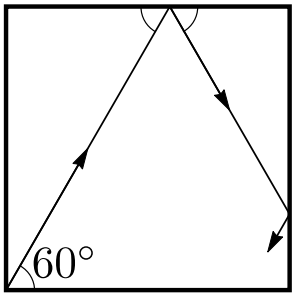
(A) A (B) B (C) C (D) D (E) E

13. Start with a big square S. Using equally spaced lines parallel to its sides, divide it into n^2 congruent small squares. If we inscribe a circle in S, what's the largest value of n for which each of these small squares includes a point inside the circle?



(A) 5 (B) 6 (C) 7
(D) 8 (E) none of these

14. A beam of light leaves a vertex of a unit square at an angle of 60° to one of the sides. Every time the beam of light hits one of the sides of the square, it reflects back into the square. What distance will the beam of light travel before it next hits a vertex?



(A) $\sqrt{5}$ (B) $2\sqrt{3}$
(C) 4 (D) $\sqrt{17}$
(E) It will never again hit a vertex.

15. Solve $a^2 - b^2 + 2bx - x^2 = 0$ for x.

(A) $x = a \pm b$ (B) $x = b \pm a$ (C) $x = \dfrac{a+b}{2} \pm \dfrac{a-b}{2}$

(D) $x = \dfrac{a}{2} \pm \dfrac{b}{2}$ (E) none of these

16. Given that $a + \ln a = 0$, what is the solution to $x^4 e^x = 256$?

(A) $x = 4a$ (B) $x = a + 4$ (C) $x = \sqrt[4]{256a}$

(D) $x = a + \ln 256$ (E) none of these

17. How many three-digit natural numbers have the property that the sum of the digits is 6?

(A) 10 (B) 19 (C) 21

(D) 28 (E) none of these

18. If you roll an ordinary six-sided die three times, what is the probability that you will get three consecutive integers in increasing order?

(A) $\dfrac{1}{216}$ (B) $\dfrac{1}{108}$ (C) $\dfrac{1}{72}$ (D) $\dfrac{1}{54}$ (E) $\dfrac{5}{216}$

19. Below are some formulas that people may *mistake* for "trig identities". However, none of these is actually true for *all* numbers A and B. Which of them is satisfied whenever $A = -B$?

(A) $\cos(AB) = A\cos(B)$

(B) $\sin(AB) = \sin(A)\sin(B)$

(C) $\cos(A + B) = \cos(A) + \cos(B)$

(D) $\sin(A) + \cos(B) = \cos(A) + \sin(B)$

(E) $\sin(A + B) = \sin(A) + \sin(B)$

20. The curve $y = x + \dfrac{3}{x+1}$ is symmetric through the point (h, k). Find $h + 3k$.

(A) -4 (B) 3 (C) -2 (D) 1 (E) 0

21. Find the range of the function $\sin(\tan^{-1} x)$

(A) $[-\pi/2, \pi/2]$ (B) $(-\pi/2, \pi/2)$ (C) $(-1, 1)$

(D) $(-1, 0) \cup (0, 1)$ (E) $[-1, 1]$

22. Which of these is a valid logical consequence of the statement "Not all flangenoids are plomental"?

(A) Some flangenoids are plomental.

(B) There are some things that are plomental but are not flangenoids.

(C) Every flangenoid is not plomental.

(D) Everything that is plomental is a flangenoid.

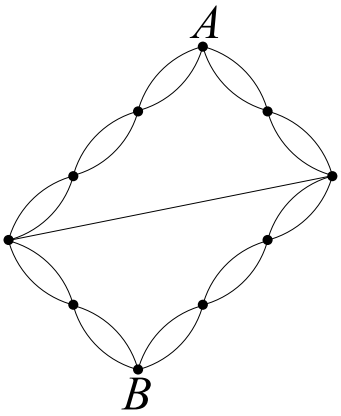
(E) There is a flangenoid which is not plomental.

23. Suppose a bacteria population grows exponentially. If the initial number of bacteria in the culture doubles after 6 hours, how long will it be before the population doubles again?

(A) another 3 hours (B) another 12 hours (C) another 9 hours

(D) another 6 hours (E) None of the above

24. In the picture to the right, each dot represents a city, and each path (curved or straight) connecting dots represents a way to get directly from one city to another. If we impose the restriction that we may visit a city at most once, how many different ways are there to get from city A to city B?



(A) 4 (B) 64 (C) 144

(D) 2048 (E) none of these

25. $\left(\cos\frac{3\pi}{10} + i\sin\frac{3\pi}{10}\right)^{10}$ equals

(A) 1 (B) -1 (C) i (D) $-i$ (E) $\frac{1+i}{\sqrt{2}}$

2016 Answers / Level 2 Test

1. A
2. B
3. D
4. E
5. D
6. E
7. E
8. B
9. E
10. E
11. A
12. B
13. B
14. E
15. B
16. A
17. C
18. D
19. E
20. A
21. C
22. E
23. D
24. C
25. B