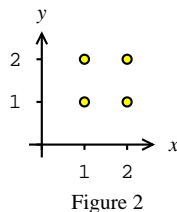
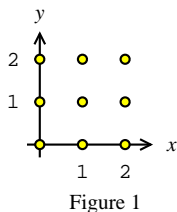


College of Charleston Math Meet 2020 Written Test – Level 2

1. How many real solutions are there to the equation $\ln(1 - x) + 4 \ln(x - 4) = 2 \ln(2 - x) + 3 \ln(x - 3)$?

(A) There are no solutions (B) 1 (C) 2
(D) 3 (E) 5

2. Due to variability in wind speed and direction, when Mary stands at the origin and throws her hat into the air, it will land at one of the 9 points in Figure 1, each with probability $1/9$. If she throws her hat into the air, walks to where it lands, and throws it into the air again, what's the probability that it will then land at one of the 4 points in Figure 2?



(A) $4/81$ (B) $9/81$ (C) $16/81$ (D) $25/81$ (E) $36/81$

3. How many solutions are there to

$$5 \sin(2x) - 8 \cos x + 5 \sin x = 4$$

in the interval $[0, 8\pi/3]$?

(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

4. Suppose that the temperature T at every point (x, y) in the plane is given by the formula

$$T = 1 - x^2 + 2y^2$$

Find the correct statement about the maximum and minimum temperature **along the line** $x + y = 1$.

(A) Min is -1 . There is no max.
(B) Max is -1 . There is no min.
(C) Max is 0 . Min is -1 .
(D) Max is 2 . Min is 0 .
(E) There is neither a max nor a min along the line.

5. A red candle is 1 inch longer than a blue candle. The red candle is lit at 4:30 and the blue candle is lit at 6:00, and at 8:30 they were the same length. The blue candle burned out at 10:00 and the red candle burned out at 10:30.

Assuming that, as each candle burns, its length decreases at a constant rate, find the original length of the blue candle.

(A) 2 inches (B) 5 inches (C) 8 inches (D) 11 inches (E) 14 inches

6. Which of the following is true about the graph of $y = \frac{1}{x-2} - \frac{4}{x^2-4}$?
- (A) There is a vertical asymptote at $x = 2$
 (B) There is a point at $x = 2$
 (C) There is neither a point nor a vertical asymptote at $x = 2$
 (D) There is a horizontal asymptote at $y = 2$
 (E) None of the others is true
7. $\tan x$ and $\tan y$ are positive integers and $\tan(x - y) = \frac{1}{4}$. What is $\tan(x + y)$?
- (A) $-\frac{8}{19}$
 (B) 4
 (C) $\frac{5}{\sqrt{17}}$
 (D) $\frac{7}{6}$
 (E) There isn't enough information to determine $\tan(x + y)$
8. There are 20 ping pong balls in a bag, each ball is stamped with an integer between 0 and 10 (0 and 10 allowed). The number stamped on each ball, if not 0, is the sum of the numbers stamped on all the other balls. Then the number of balls stamped with 0 is:
- (A) no more than 5 (B) 10 (C) 13
 (D) 16 (E) at least 18
9. You have three boxes of unknown weight and a scale which only works for items that weigh more than 100 pounds. Since the boxes each weigh less than 100 pounds alone, you decide to weigh them together in pairs. The three different pairs weigh 113, 116 and 117 pounds respectively. By how many pounds does the weight of the heaviest box exceed the weight of the next heaviest box?
- (A) 1 pound (B) 2 pounds
 (C) 3 pounds (D) 4 pounds
 (E) cannot tell from information given
10. Let ρ be a rotation about the origin by 51° clockwise. What is the minimal number of times one must repeat ρ before every point returns to where it started?
- (A) 30 (B) 60 (C) 120
 (D) 360 (E) none of these
11. When I write a certain number in base n , it looks like 25. When I write *twice* that number in base n , it looks like 52. Which of these sets *contains* the number n ?
- (A) {6, 11, 20} (B) {7, 10, 14} (C) {12, 13, 18}
 (D) {8, 15, 19} (E) {5, 9, 16}

12. Find the distance from the point $(7, 9)$ to the line $x - y = 0$.
- (A) 1 (B) $\sqrt{2}$ (C) 2
 (D) 10 (E) none of these
13. Evanium is a radioactive substance which decays exponentially with time. In 2 hours, a sample of Evanium will decay to 90% of its original mass. How many hours will it take for a sample to decay to 10% of its original mass?
- (A) 18 (B) $\frac{2}{1 - \log 9}$ (C) $\frac{\ln 10}{\ln 10 - \ln 9}$
 (D) $2 \ln \left(\frac{1}{9}\right)$ (E) $-\log_{9/10} 10$
14. Six balls numbered 1 through 6 are randomly dropped into six boxes numbered 1 through 6. That is, one ball is dropped in each box, with an equal probability that any given ball ends up in any given box. We say that a ball is in "the right box" if it has the same number as the box that it is in. Which of these events has the *lowest* probability?
- (A) no ball ends up in the right box
 (B) exactly one ball ends up in the right box
 (C) exactly three balls end up in the right boxes
 (D) exactly five balls end up in the right boxes
 (E) all of the balls end up in the right boxes
15. Find the constant term in the expansion
- $$\left(x - \frac{1}{x}\right)^6 = x^6 + \dots + \frac{1}{x^6}$$
- (A) 20 (B) -20 (C) 16 (D) -15 (E) -15
16. Ned says, "A year from now, I will be 4 times as old as my sister was when my brother was born. A year after that, my brother will be 4 times as old as I was when my sister was born. That year, my sister will be 7 times as old as our dog Frieda." All the ages are positive integers. How old is Ned's brother?
- (A) Not enough information is available. (B) 6
 (C) 18 (D) 30
 (E) Ned is lying.
17. Evaluate $\cos^{-1}\left(-\frac{3}{5}\right) - \sin^{-1}\left(-\frac{4}{5}\right)$ in radians.
- (A) 0 (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$
 (D) π (E) None of these

18. Which of these is a solution to the equation

$$2^{2x} - 8 \cdot 2^x = -12?$$

- (A) $1 + \frac{\log 3}{\log 2}$ (B) $\frac{1}{2} \log 6$ (C) $1 + \log \frac{3}{2}$
(D) $\log 3$ (E) none of these

19. A is the set $\{1, 2, 3, 4\}$. B and C are nonempty sets. $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B \cap C = \emptyset$. If the number of elements of B is greater than the number of elements in C , find the smallest possible sum of the elements of B .

- (A) 6 (B) 8 (C) 10 (D) 11 (E) 13

20. A certain state's license plate contains 3 numbers followed by 3 letters. If no license plate can start with a zero, what is the probability that the letter portion of a randomly selected plate is the word DOG?

- (A) $\frac{900}{10^3 * 26^3}$ (B) $\frac{1}{26^3}$ (C) $\frac{999}{10^3 * 26^3}$ (D) $\frac{3}{900 * 26^3}$ (E) $\frac{1}{900 * 26^3}$

21. In the complex plane, there is a straight line that contains all six roots of $z^6 - 8iz^4 - 19z^2 + 12i = 0$. Which of the following points is on that line?

- (A) $1 + i$ (B) 1 (C) i
(D) $1 + 2i$ (E) None of these

22. A sphere is placed on a horizontal plane during a sunny day. At a certain instant of time, its shadow reaches 10 meters from the point where the sphere touches the plane. At the same time, a 1 meter tall post casts a 2 meter long shadow. What is the radius of the sphere, expressed in meters?

- (A) $\frac{5}{2}$ (B) $9 - 4\sqrt{5}$ (C) $10\sqrt{5} - 20$ (D) $8\sqrt{10} - 23$ (E) $6 - \sqrt{15}$

23. In a given forest, suppose there are more trees than there are leaves on any *single* tree. Which of the following must be true?

- I. There exist two trees with a leaf in common.
II. There exist at least two trees which have the same number of leaves.
III. There exists a tree with no leaves.

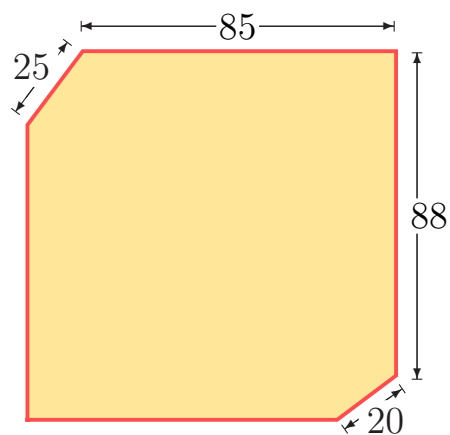
- (A) I only (B) II only (C) III only (D) either II or III (E) either I or III

24. Let $f(x) = \alpha x(x - 1)(x - 2) + \beta x(x - 1) + \gamma x + \delta$.

Find α if $f(0) = f(1) = f(2) - 1 = f(3) - 1$.

- (A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{3}$ (E) $-\frac{1}{6}$

25. Two corners were cut off a 100×100 square resulting in the hexagon shown in the figure. Four of the sides have the lengths indicated by the labels. What is the total perimeter of this shape?



- (A) 368 (B) 372 (C) 378 (D) 382 (E) 388

2020 Answers / Level 2 Test

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|------|-------|-------|
| 1. A | 10. C | 19. B |
| 2. D | 11. D | 20. B |
| 3. B | 12. B | 21. A |
| 4. A | 13. B | 22. C |
| 5. C | 14. D | 23. D |
| 6. C | 15. B | 24. D |
| 7. A | 16. C | 25. D |
| 8. E | 17. D | |
| 9. C | 18. A | |