

College of Charleston
Math Meet 2022
Written Test – Level 2

- Suppose $p(x)$ is a polynomial of degree 5, and that
when $p(x)$ is divided by $(x - 2)(x - 3)$, the remainder is $4 - x$,
when $p(x)$ is divided by $(x - 1)(x - 3)$, the remainder is $x - 2$, and
when $p(x)$ is divided by $(x - 1)(x - 2)$, the remainder is $3x - 4$.
Find the remainder when $p(x)$ is divided by $(x - 1)(x - 2)(x - 3)$.
(A) $2x^2 - 11x + 16$
(B) $-2x^2 + 9x - 8$
(C) $2x^2 - 7x + 4$
(D) $-2x^2 + 5x - 2$
(E) Impossible. There is no such polynomial p .
- How many subsets of $\{1, 2, \dots, 100\}$ are there so that the sum of their elements is 5042?
(A) 3 (B) 4 (C) 6 (D) 7 (E) 10
- What is the coefficient of x^{8n} in $x^4(2x^2 + 3)^{4n}$?
(A) $n6^{3n}$ (B) $(8n)3^{4n-1}$ (C) $216n$
(D) $2^{4n-1}(9n)(4n - 1)$ (E) None of these
- How many fair six-sided dice must you roll in order to have at least a 50% chance of two or more matching dice?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6
- Which of the following is **not** equivalent to the statement “It rhymes when it runs and rains.”?
(A) It rhymes or doesn’t run or doesn’t rain.
(B) If it rains but not rhymes, then it doesn’t run.
(C) When it runs but not rhymes, it doesn’t rain.
(D) If it runs, then it rhymes or doesn’t rain.
(E) When it rains but not runs, it doesn’t rhyme.
- Five different numbers are chosen at random from
 $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
If any set of five numbers has the same probability of being selected, what’s the probability that their median is 3?
(A) $5/28$ (B) $1/63$ (C) $1/3$ (D) $8/27$ (E) $1/72$
- Six identical chocolates and five identical cookies are to be distributed (not necessarily evenly) among Amy, Bill, and Carrie. If Amy must receive at least two chocolates and Bill must receive at least one cookie, in how many ways can the chocolates and cookies be distributed? It’s possible that some of the people receive no chocolates at all or no cookies at all.
(A) 225 (B) 302 (C) 360
(D) 588 (E) None of these

8. Which of the following functions satisfies $f(f(x)) = x$ (on the domain of $f \circ f$)?
- (A) $f(x) = 1 - x^2$ (B) $f(x) = \sqrt{1 - x}$ (C) $f(x) = \sin(x)$
- (D) $f(x) = 2^{1/x}$ (E) $f(x) = \frac{-x + 2}{2x + 1}$

9. In the complex plane, $|z - 3| + |z + 3| = 10$ is the equation of a curve. If we regard the same curve as lying in the xy -plane, what is its equation?
- (A) $\frac{x^2}{25} - \frac{y^2}{9} = 1$ (B) $y = x^2 - 25$ (C) $\frac{x^2}{25} + \frac{y^2}{16} = 1$
- (D) $x^2 + 3y^2 = 25$ (E) None of these

10. Suppose that we choose a random value θ in the interval $[0, 2\pi]$. Which of the following is most likely to happen?
- (A) $-\cos(\theta) > 0 > -\sin(\theta)$
- (B) $\sin(\theta) > \cos(\theta) > -\cos(\theta)$
- (C) $\cos(\theta) > \sin(\theta) > -\sin(\theta)$
- (D) $\sin(\theta) > \cos(\theta) > -\sin(\theta) > -\cos(\theta)$
- (E) $\sin(\theta) > \cos(\theta) > 0$

11. I have an original idea for a game, in which the player must guess a passcode consisting of 5 distinct digits (0,1,2,..., 9). After each 5-digit guess, the game will give the player some clues as to which digits in her guess appear in the correct answer. See figure. If a player makes the two guesses shown at the bottom of the figure, what is the remainder of the correct answer when it is divided by 9?

2 A digit on a WHITE background appears in this place in the correct answer.

1 A digit on a GRAY background appears in the correct answer, but not in this place.

0 A digit on a BLACK background does not appear in the correct answer.

6	9	1	8	7
4	1	0	2	5

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
12. A sample of bacteria in a Petri dish (labeled "X") grows exponentially, meaning that its population at time t equals $C \cdot a^t$ for some constants a and C . Bacteria in a second dish (labeled "Y") also exhibit exponential growth. The doubling time of population Y (that is, the length of time required for the population to double) is twice that of population X. At time 2, population X equals what population Y was at time 0. At time 8, population X is 8 times population Y. At what time were the two populations the same?
- (A) $\log_3 81$ (B) $2^{3/2}$ (C) $2^{5/2}$
- (D) $\frac{\log_2(16)}{\log_2(2\sqrt{2})}$ (E) Not enough information.

13. x_n is a binary number with $2n$ bits, half of which are 0 and half of which are 1. As a function of n , what's the largest possible value for x_n ?
- (A) $17n^2 - 41n + 26$ (B) 2^n (C) $(2n - 1)2^n$
 (D) $(2^n - 1)2^n$ (E) None of these
14. A biased coin has a probability p of coming up heads, where p is a rational number that's less than $\frac{1}{2}$. If the probability of two consecutive flips of the coin being identical is $\frac{145}{288}$, what's the probability of getting two heads in a row?
- (A) $\frac{5}{24}$ (B) $\frac{1}{144}$ (C) $\frac{71}{288}$
 (D) $\frac{121}{576}$ (E) None of these
15. Suppose that the highest point on the graph of $y = f(x)$ is $(3, 1)$ and that the lowest point is $(5, -3)$. What is the highest point on the graph of $y = 3(f(2x - 1))^2 + 6$?
- (A) $(3, 33)$ (B) $(2, 9)$ (C) $(5, 9)$
 (D) $(9, 33)$ (E) None of these
16. Let A be the point $(1, 3)$. Let B be the point $(5, 11)$. $\triangle ABC$ is a right triangle with right angle at C . What is the largest that the area of $\triangle ABC$ could be?
- (A) $10\sqrt{3}$ (B) $20\sqrt{3}$ (C) 40
 (D) 80 (E) none of these
17. Which of the following has the most solutions in the interval $[0, 2\pi]$?
- (A) $\cos(100x) = 0$ (B) $\sin(100x) = 0$ (C) $\cos(101x) = 0$
 (D) $\cos(200x) = 1$ (E) $\sin(200x) = 1$
18. Which of the following never equals $\ln\left(\frac{x}{x-100}\right)^2$ on any open interval?
- (A) $2\ln(x) - 2\ln(x-100)$ (B) $2\ln(x) - 2\ln(100-x)$
 (C) $2\ln(-x) - 2\ln(100-x)$ (D) $2\ln(-x) - 2\ln(x-100)$
 (E) None of these
19. Suppose that we rotate the xy -plane 45° counterclockwise around the origin and then rotate 45° clockwise about the point $(1, 1)$. Which of the following describes the overall motion that has been performed?
- (A) A translation in the direction of the y -axis (positive or negative)
 (B) A translation in some direction other than that of the y -axis
 (C) A rotation by 45° (clockwise or counterclockwise) around some point
 (D) A rotation by some amount other than 45° around some point
 (E) None of the these

20. If x is an integer, which of the following must be a divisor of $x^{320} - x^{170} - x^{151} + x$?
 (A) 4 (B) 13 (C) 17
 (D) 31 (E) None of these
21. Let C be a circle of radius 4 with center O . Let A and B be points on C so that segment AB has length 3. Let P be the point on C that maximizes the area of triangle $\triangle ABP$. How is the area of $\triangle ABP$ related to the area of $\triangle ABO$?
 (A) It's double (B) It's 12 more
 (C) It's triple (D) It's 6 more
 (E) None of the others must be true
22. A shape in three-dimensional space is first rotated by 180° around the y -axis and then rotated by 180° around the x -axis. Which of the following actions will return the shape to its original position?
 (A) Rotate 180° around z -axis (B) Translate 1 unit in positive x -direction
 (C) Reflect through the plane $y = z$ (D) Reflect through plane $x = 0$
 (E) None of these

23. Let $p(x)$ and $q(x)$ be polynomials of degree 6 and 7 respectively. If

$$(x^3 + 7)p(x) + (x^2 + 4)q(x) = (x^2 - 1)$$

which of the following must be true?

- (A) The common roots of $p(x) = 0$ and $q(x) = 0$ are at least $x = \pm 1$
 (B) The common roots of $p(x) = 0$ and $q(x) = 0$ are exactly $x = \pm 1$
 (C) The common roots of $p(x) = 0$ and $q(x) = 0$ are at most $x = \pm 1$
 (D) $x = 1$ or $x = -1$ is a common root of $p(x) = 0$ and $q(x) = 0$
 (E) None of the others must be true
24. If n is a positive integer, what is the largest solution to $\cos(nx) = \frac{1}{2}$ for $0 \leq x \leq 2\pi$?
 (A) $\pi + \frac{2n\pi}{3}$ (B) $\frac{5\pi}{3n}$ (C) $2\pi - \frac{n\pi}{3}$
 (D) $2\pi - \frac{\pi}{3n}$ (E) None of these
25. Circle C_1 has center P_1 and radius 10. Circle C_2 has center P_2 and radius 2. The distance from P_1 to P_2 is 24. A line ℓ is tangent to C_1 at A_1 and tangent to C_2 at A_2 . The line ℓ intersects the (extended) line P_1P_2 at B (which isn't between P_1 and P_2). What is the area of triangle A_1BP_1 ?
 (A) 120 (B) 137.5 (C) 180
 (D) $100\sqrt{2}$ (E) None of these

2022 Answers / Level 2 Test

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|------|------|------|
| 1. B | 4. C | 7. A |
| 2. C | 5. E | 8. E |
| 3. D | 6. A | 9. C |

- 10. A
- 11. E
- 12. A
- 13. D
- 14. D
- 15. A

- 16. E
- 17. C
- 18. D
- 19. B
- 20. D
- 21. D

- 22. A
- 23. C
- 24. D
- 25. D