College of Charleston
Math Meet 2023
Written Test – Level 2

1. Solve $\log_x 2 + \log_x 3 = \frac{1}{2}$.

   (A) $x = 5$ (B) $x = \sqrt{\frac{3}{2}}$ (C) $x = 6$

   (D) $x = 36$ (E) None of these

2. Which of the following is equal to $3^{4\log_2 5}$?

   (A) $3^{5\log_2 4}$ (B) $4^{5\log_2 3}$ (C) $5^{4\log_2 3}$

   (D) $5^{3\log_2 4}$ (E) None of these

3. How many solutions to $2 - 5\sin x \cos x = \sqrt{5}(2\sin x - \cos x)$ are in the interval $[0, 2\pi]$?

   (A) none (B) 1 (C) 2 (D) 3 (E) 4

4. The polynomial $p(x) = x^4 - x^2 + ax + b$ can be factored into the product of two polynomials, one of which is $x^2 - 4$. What is $a + b$?

   (A) 2 (B) -8 (C) 3 (D) -12 (E) 4

5. Let C be the curve defined parametrically by $x = \sin(t), y = \cos(2t)$. Which of the following equations represents the same curve?

   (A) $y = x^2 - 2x$ for $-1 \le x \le 1$

   (B) $y = x^2 - 1$ for $0 \le x \le 1$

   (C) $y = 1 - 2x^2$ for $-1 \le x \le 1$

   (D) $y = \sin(x) - \cos(x)$ for $0 \le x \le \frac{\pi}{2}$

   (E) None of these

6. Let x and y be the binary numbers $x = (10000001)_2$ and $y = (10010011)_2$. What is the product $x \cdot y$ in binary?

   (A) $(100001110010011)_2$ (B) $(100100000010011)_2$ (C) $(100100110010011)_2$

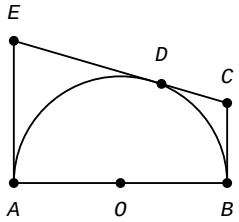
   (D) $(10010010110011)_2$ (E) None of these

7. The figure shows the result of shuffling a deck of 26 cards perfectly one time. How many *additional* perfect shuffles must we perform to return the cards to their original position?

original position:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
after one shuffle:	1	14	2	15	3	16	4	17	5	18	6	19	7	20	8	21	9	22	10	23	11	24	12	25	13	26

   (A) 19 (B) 21 (C) 23 (D) 25 (E) 27

8. In the accompanying figure (not to scale), $AOBDA$ is a semicircle centered at O. The lines BC, CE, and EA are tangent to the semicircle at B, D, and A, respectively. If the lengths $\overline{AO} = 1$ and $\overline{AE} = \dfrac{5}{4}$, what is the area of the trapezoid $ABCE$?
In the correct answer, the second digit after the decimal is



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

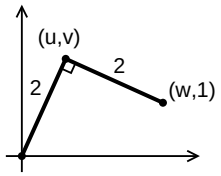
9. A pair of ants are inside a cubical cardboard box with sides 1 meter long. One ant is at the center of the top of the box, and the other is on the bottom, $\frac{1}{4}$ meter from one side and $\frac{1}{8}$ meter from another side.
Assuming both ants stay on the cardboard, how many meters must one crawl to reach the other?
(A) $\frac{1}{8}\sqrt{169}$ (B) $\frac{1}{8}\sqrt{173}$ (C) $\frac{1}{8}\sqrt{187}$ (D) $\frac{1}{8}\sqrt{205}$ (E) $\frac{1}{8}\sqrt{295}$

10. The nation of Torf consists entirely of truth-tellers and liars. Truth-tellers tell the truth 90% of the time, and liars tell lies 60% of the time. But if you ask everyone in Torf whether they're truth-tellers or liars, the number of reported liars will equal the number of actual liars. What part of the population of Torf are truth-tellers?
When the answer is expressed as a fraction between 0 and 1 in lowest terms, the denominator minus the numerator equals
(A) 7 (B) 5 (C) 4 (D) 3 (E) 1

11. a and b are real numbers such that $(a+bi)^2 = 11 - 60i$. What is $(a+b)^2$?
(A) 1 (B) 13 (C) 16
(D) 121 (E) None of these

12. In the interval $[0, 2\pi]$, how many solutions are there to the equation $\cos 2x + \cos 3x = 0$?
(A) none (B) 1 (C) 2 (D) 3 (E) 5

13. Two orthogonal line segments each of length 2 lie in the first quadrant as shown. Find the coordinate v.



(A) $\dfrac{1}{2}(\sqrt{7}+1)$ (B) $\dfrac{1}{2}(\sqrt{7}-2)$ (C) $\dfrac{1}{2}(\sqrt{5}+3)$ (D) $\dfrac{1}{2}(\sqrt{3}+2)$ (E) $\dfrac{1}{2}(\sqrt{5}-1)$

14. A logical statement can be thought of as a variable that can take only the values true (T) or false (F). The operators $\wedge$ ("and"), $\vee$ ("or"), $\Rightarrow$ ("implies"), and $\neg$ ("not") act on logical statements to produce other statements. For instance, $p \vee q$ is T when either p or q (or both) is T. $p \Rightarrow q$ is T except when p is T and q is F.
If $(p \wedge \neg q) \Rightarrow (r \vee \neg s)$ is false, how many of the following statements must be true?
I. $(q \wedge r) \Rightarrow (p \wedge s)$
II. $p \wedge (\neg q) \wedge (\neg r) \wedge s$
III. $p \vee q \vee (\neg r) \vee (\neg s)$
IV. $(r \vee \neg s) \Rightarrow (p \wedge \neg q)$ is also false.
V. $p \vee (r \wedge s) \Rightarrow (p \vee r) \wedge (p \vee s)$.
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. Find the number of real solutions to

$$x^4 - \sqrt{2}x^3 + 2x^2 - 2\sqrt{2}x + 4 = 0$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. What's the smallest $n > 1$ such that $n!$ is divisible by n^4?

(A) 9 (B) 15 (C) 18

(D) 20 (E) None of these

17. Let $[\![x]\!]$ denote the greatest integer less than or equal to x. How large must x be to ensure that $\dfrac{x}{[\![x]\!]}$ is less than $.001$ unit away from 1?

(A) $x \geq 100$ (B) $x \geq 500$ (C) $x \geq 999$ (D) $x \geq 1000$ (E) $x \geq 1001$.

18. A **code** consists of a sequence of 10 digits with repetitions allowed, for example, 0034768927. Determine the probability that the sequence **01234** will occur consecutively in a randomly generated code, for example, as it does in 847**01234**62 but does not in 82**01236**499.

(A) 5.00001×10^{-5} (B) 5.0×10^{-5} (C) 6.0×10^{-5}

(D) 6.0×10^{-10} (E) 5.99999×10^{-5}

19. The Cougars, the College of Charleston Men's Basketball team, has a roster including 6 forwards, 7 guards, and one center. At any time during a game, there will be two guards, two forwards and one center on the floor. How many different line-ups can the Cougars put on the floor during a game assuming that there is no distinction made between the two guard positions and similarly no distinction between the two forward positions?

(A) 42 (B) 315 (C) 2,427

(D) 35,170 (E) more than 1 million

20. Find the polar coordinates (r, θ) of the point which does **not** belong to the graph of the polar equation $r = \sin 2\theta$.

(A) $\left(\dfrac{1}{2}, \dfrac{\pi}{12}\right)$ (B) $\left(1, \dfrac{\pi}{4}\right)$ (C) $\left(0, \dfrac{\pi}{4}\right)$ (D) $\left(\dfrac{1}{2}, \dfrac{\pi}{3}\right)$ (E) $\left(\dfrac{1}{2}, \dfrac{19\pi}{12}\right)$

21. Which of the following is the reflection of $y = \sin(x)$ around the line $x = \dfrac{\pi}{3}$?

(A) $y = -\dfrac{1}{2}\sin(x) - \dfrac{\sqrt{3}}{2}\cos(x)$ (B) $y = \dfrac{1}{2}\sin(x) + \dfrac{\sqrt{3}}{2}\cos(x)$

(C) $y = 2\sin(x)\cos(x)$ (D) $y = \dfrac{\sqrt{3}}{2}\cos(x) - \dfrac{1}{2}\sin(x)$

(E) None of these

22. Which of the triples of numbers listed below are the lengths of the edges of a right triangle with perimeter equal to 1000?

(A) $(252, 336, 420)$ (B) $(222, 373, 405)$ (C) $(202, 372, 426)$

(D) $(250, 338, 420)$ (E) $(200, 375, 425)$

23. Let D be the circle $x^2 + 2x + y^2 = 10$. No point on D has integer coordinates (meaning x and y can't both be integers). What is the distance from D to the nearest point with integer coordinates?

(A) $\sqrt{16} - \sqrt{11}$ (B) $\sqrt{11} - \sqrt{8}$ (C) $\sqrt{13} - \sqrt{11}$

(D) $\sqrt{11} - \sqrt{9}$ (E) none of the above.

24. Austin's Famous boxed chocolates come in boxes of 6, 9, and 20 pieces. So, you *can* buy 15 pieces (by buying a box of 6 and a box of 9), but it is not possible to buy exactly 16 pieces. The largest number of pieces that you cannot buy is a two digit number. Give the sum of its digits.

(A) 7 (B) 8 (C) 10

(D) 11 (E) none of these

25. For any two real numbers a and b, define their "diamond product" to be

$$a \diamond b = 4a^2 + 4b^2 - a^2b^2.$$

Which of the following statements are true?

I. The diamond product satisfies the law of commutativity.
II. The diamond product satisfies the distributive law (with respect to ordinary addition of numbers).
III. The diamond product satisfies the associative law.
IV. The equation $c \diamond x = 4c^2$ has one solution for most values of c, but infinitely many for others.
V. There are always exactly two solutions to the equation $c \diamond x = c$ no matter what number c is.

(A) Only I, II and III are true. (B) Only II and V are true.

(C) Only I and IV are true. (D) All of the statements are true.

(E) None of the statements are true.

2023 Answers / Level 2 Test

1. D
2. C
3. D
4. D
5. C
6. E
7. A
8. E
9. B
10. E
11. A
12. E
13. A
14. E
15. A
16. E
17. D
18. E
19. B
20. D
21. B
22. E
23. E
24. A
25. C