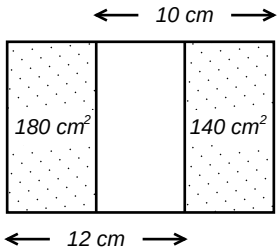
College of Charleston
Math Meet 2024
Written Test – Level 2

1. Two horizontal and four vertical lines form three rectangles as shown. (Figure is not to scale.) Find the area in cm^2 of the middle, unshaded rectangle.



(A) 60 (B) 70 (C) 80 (D) 90 (E) 100

2. Find the number of integer solutions (x, y) to

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$$

if, for instance, $(x, y) = (2, -4)$ and $(x, y) = (-4, 2)$ are counted as different integer solutions.

(A) 12 (B) 10 (C) 9 (D) 8 (E) 6

3. Find the remainder when

$$\big(t^2 + 9 + (t^3 - 4t)(t^4 + 5t^3 - 4t^2 - 20t)\big)\big(t - 3 + (2t^2 - 8)(t^3 - 9t^2 + 3t - 27)\big)$$

is divided by $t^2 - 4$.

(A) $15t + 20$ (B) $13t - 39$ (C) $-8t - 28$ (D) $-11t + 13$ (E) 111

4. Find the 93rd digit after the decimal point in the decimal form of $\dfrac{1,234}{99,999}$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

5. Let A be the point $(3, 4)$, B be the point $(5, 8)$, C be the point $(-1, -1)$, D be the point $(0, 1)$, E be the point $(1, 3)$, and F be the point $(2, 5)$. Among the triangles ΔABC, ΔABD, ΔABE, and ΔABF, which has the largest area?

(A) ΔABC (B) ΔABD (C) ΔABE
(D) ΔABF (E) They're all the same area

6. $\overline{ABCD}$ is a four-sided polygon. No two edges of $\overline{ABCD}$ cross. Sides $\overline{AB}$ and $\overline{CD}$ are parallel. Find the area of $\overline{ABCD}$ if the lengths of its sides are

$$AB = 10 \qquad BC = 13 \qquad CD = 6 \qquad DA = 15$$

(A) 80 (B) 84 (C) 88 (D) 92 (E) 96

7. To win at a certain game, you must toss a coin 5 times and get "heads" each time. If you lose the game, what's the probability that you lost on the first toss?
Assume that each time you toss the coin, there's a 50% probability that you'll get "heads."

(A) $\dfrac{32}{63}$ (B) $\dfrac{16}{31}$ (C) $\dfrac{9}{16}$ (D) $\dfrac{3}{7}$ (E) $\dfrac{1}{2}$

8. If $0 \leq a \leq 1$, what is the maximum value of the function $a \sin x + \sqrt{1-a^2} \cos x$?

(A) a (B) $a + \sqrt{1-a^2}$

(C) $\sqrt{a^2(1-a^2)}$ (D) The larger of a and $\sqrt{1-a^2}$

(E) 1

9. If x is a complex number for which $x + \dfrac{1}{x} = 1$, find $x^5 + \dfrac{1}{x^5}$.

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

10. If x is the number

$$x = (\log_8 2)^{(\log_2 8)}$$

then what is $\log_3 x$?

(A) -3 (B) $-\dfrac{1}{3}$ (C) $\dfrac{1}{3}$ (D) 3 (E) 9

11.

$$\frac{1}{2i}\left(\left(\frac{1+i}{\sqrt{2}}\right)^{10} - \left(\frac{1-i}{\sqrt{2}}\right)^{10}\right) =$$

(A) 1 (B) -1 (C) 0 (D) i (E) $-i$

12. Suppose that $f(x) = \dfrac{ax+b}{cx+d}$ satisfies the following conditions:

- $y = f(x)$ has a vertical asymptote at $x = -1$
- $y = f(x)$ has a horizontal asymptote at $y = -\dfrac{1}{3}$
- $f(4) = 0$

What is $f(-2)$?

(A) -2

(B) 1

(C) 2

(D) $\dfrac{1}{3}$

(E) there isn't enough information to determine $f(-2)$.

13. The function $2e^x \sin x - 2e^{-x} \cos x$ can be written as the sum of an even function and an odd function. Find the odd function.

(A) $(e^x - e^{-x}) \cos x + (e^x - e^{-x}) \sin x$

(B) $(e^x + e^{-x}) \cos x - (e^x + e^{-x}) \sin x$

(C) $(e^x + e^{-x}) \cos x + (e^x - e^{-x}) \sin x$

(D) $(e^x - e^{-x}) \cos x + (e^x + e^{-x}) \sin x$

(E) $(e^x - e^{-x}) \cos x - (e^x + e^{-x}) \sin x$

14. How many real solutions are there to $\tan^{-1}(\tan x) = \dfrac{\pi}{2} - x$?

(A) there are no real solutions (B) 1

(C) 2 (D) 5

(E) infinitely many

15. It takes Bao 1 hour to walk to school from his home. On Monday, he started walking to school without his homework. His mother realized this and began walking to school with the homework 20 minutes after he left the house. She caught up with him, gave him his homework and then immediately walked home, arriving there 25 minutes after she started.
Assuming they both walked at a constant speed without stopping, how many hours would it take Bao's mother to walk to school from their home?

(A) $\dfrac{2}{3}$ (B) $\dfrac{5}{7}$ (C) $\dfrac{5}{13}$ (D) $\dfrac{3}{4}$ (E) $\dfrac{7}{9}$

16. I hold 5 gold coins in my right hand and 7 silver coins in my left. The coins in my right hand weigh 4 ounces more than the coins in my left. After I move one gold coin from my right to my left and one silver coin from my left to my right, the coins in my right hand weigh 12 ounces less that those in my left.
Assuming each gold coin has the same weight, and each silver coin has the same weight, find the combined weight in ounces of 1 gold and 2 silver coins.

(A) 62 (B) 60 (C) 59 (D) 45 (E) 42

17. Let p be the polynomial

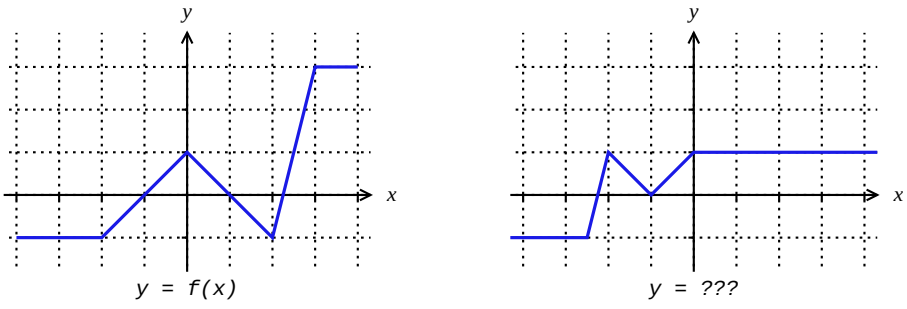
$$p(x) = (x^2 - 0^2)(x^2 - 1^2)(x^2 - 2^2)\cdots(x^2 - 11^2)$$

It happens to be the case that no matter what integer x is, $p(x)$ is always divisible by 2, by 3, and by lot of other numbers. What's the smallest positive integer n such that there's an integer x for which $p(x)$ *isn't* divisible by n?

(A) 13 (B) 16 (C) 23

(D) 29 (E) none of these

18. The graph of $f(x)$ appears on the left. What function is graphed on the right?



(A) $\dfrac{1}{2} + \dfrac{1}{2}f(2x-2)$ (B) $\dfrac{1}{2} - \dfrac{1}{2}f(-2x-2)$ (C) $\dfrac{1}{2} + \dfrac{1}{2}f(\dfrac{1}{2}x - \dfrac{1}{2})$

(D) $\dfrac{1}{2} - \dfrac{1}{2}f(-\dfrac{1}{2}x+1)$ (E) $\dfrac{1}{2} + \dfrac{1}{2}f(2x+1)$

19. Find $p(1)$ if $p(x)$ is a polynomial and

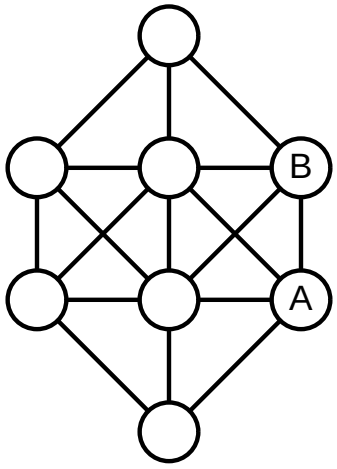
$$p(x+1) - p(x-1) = 12x - 8$$
$$p(x+i) + p(x-i) = 6x^2 - 8x + 6 \qquad (1)$$

(A) 5 (B) 9 (C) -13
(D) $1 - i$ (E) not enough information

20. An ATM (an automated banking machine) is able to distribute five dollar bills, ten dollar bills and twenty dollar bills. By mistake, an employee puts twenty dollar bills in the five dollar bill slot. As a result, some people get more money out of the machine than they *should* when they make a withdrawal. Which of these amounts could possibly be the amount of money that the bank lost due to this mistake?

(A) $\$9223$ (B) $\$101$ (C) $\$99$ (D) $\$315$ (E) $\$522$

21. Each of the eight circles in the figure is to be filled with exactly one of the digits $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Each digit in this list is to be used exactly once. No two adjacent numbers in this list (2 and 3, for example) may go in circles that are connected by a line. For instance, if A is 2, then B cannot be 1 or 3. (The numbers 1 and 8 are not considered adjacent.) Find B if A is 3.



(A) 1 (B) 5 (C) 6 (D) 7 (E) 8

22. A bug starts at the origin of the xy-plane. Every second it goes up or right one unit, each with probability 0.5. At one instant, you notice that the bug is at the point (3,4). What is the probability that the bug went through the point (3,3)?

(A) $\frac{3}{5}$ (B) $\frac{1}{2}$ (C) $\frac{3}{7}$ (D) $\frac{2}{5}$ (E) $\frac{4}{7}$

23. A pyramid has its peak directly over the center of its base, which is a square with side length 10 cm. If its height is 5 cm, what is the surface area of the pyramid in cm^2?

(A) $25\sqrt{2}$ (B) $50\sqrt{2}$ (C) $100\sqrt{2}$
(D) $100 + 100\sqrt{2}$ (E) $100 + 200\sqrt{2}$

24. Suppose that we have the following system of equations:

$$\begin{aligned} a + b &= -1 \\ a\cos(\theta) + b\cos(\psi) &= 1 \\ a\cos^2(\theta) + b\cos^2(\psi) &= 1 \end{aligned}$$

What is $a\cos(2\theta) + b\cos(2\psi)$?

(A) -1

(B) 0

(C) 1

(D) 3

(E) the value can't be determined without more information

25. $\frac{10}{1} + \frac{9}{2} + \frac{8}{4} + \frac{7}{8} + \cdots + \frac{1}{512}$ is very close to being 18. How far is it from 18?

(A) $\frac{1}{1024}$ (B) $\frac{1}{512}$ (C) $\frac{3}{1024}$ (D) $\frac{3}{512}$ (E) none of these

2024 Answers / Level 2 Test

1. A
2. C
3. B
4. C
5. E
6. E
7. B
8. E
9. D
10. A
11. A
12. A
13. D
14. C
15. C
16. A
17. D
18. B
19. A
20. D
21. B
22. E
23. D
24. D
25. B