College of Charleston
Math Meet 2004
Written Test – Level 3

1. Suppose that X and Y represent two numbers chosen independently and at random from the interval $(-1, 1)$. What is the probability that $|X + Y| > 1$?

   (A) 1/4 (B) 1/2 (C) 3/4 (D) 1/3 (E) 2/3

2. If m and n are two positive integers such that $\log_{10} m = 12.3...$ and $\log_{10} n = 15.4...$, how many digits are there in the decimal expansion of the product $m \cdot n$?

   (A) 3 (B) 16 (C) 27 (D) 28 (E) 189

3. Three circles pass through the origin of a Cartesian plane. The center of the first circle belongs to the first quadrant, the center of the second circle belongs to the second quadrant, and the center of the third circle belongs to the third quadrant. Let P be the intersection of the interiors of the three circles. Find the true statement.

   (A) P must be the empty set.

   (B) P may be nonempty, in which case it must be a subset of quadrant one.

   (C) P may be nonempty, in which case it must be a subset of quadrant two.

   (D) P may be nonempty, in which case it must be a subset of quadrant one or three.

   (E) P may be nonempty, in which case it must be a subset of quadrant four.

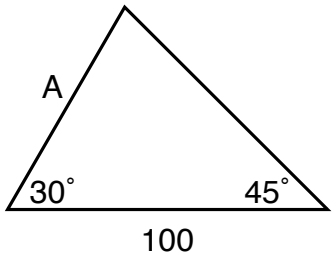
4. The keys in a square keypad of side 4 are numbered 1 through 16. Secret agent 007 must press two keys simultaneously in order to access the enemy headquarters. If he presses the wrong two keys, an alarm will sound. The only piece of information he knows is that the two keys are not adjacent (i.e., they have no side or vertex in common). What is the probability that he will guess the right combination and gain access to the enemy headquarters?

   (A) $\frac{1}{64}$ (B) $\frac{1}{78}$ (C) $\frac{1}{128}$ (D) $\frac{1}{156}$ (E) $\frac{1}{160}$

5. If b is a real number, and if the average rate of change of the function $f(x) = b^x$ on the interval $[2, 4]$ is 3, what is its average rate of change on $[4, 6]$?

   (A) 3 (B) 4 (C) 4.5 (D) 6 (E) 9

6. Find the length of side A in the triangle.



   (A) $200/\sqrt{3}$ (B) $100(\sqrt{3} - 1)$ (C) $100\sqrt{3}$

   (D) 50 (E) $100/\sqrt{2}$

7. Find the polar coordinates (r, θ) of the point which does **not** belong to the graph of the polar equation $r = \sin 2\theta$.

(A) $\left(\frac{1}{2}, \frac{\pi}{12}\right)$ (B) $\left(1, \frac{\pi}{4}\right)$ (C) $\left(0, \frac{\pi}{4}\right)$

(D) $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ (E) $\left(\frac{1}{2}, \frac{19\pi}{12}\right)$

8. Find $p(-1)$ if p is the cubic polynomial which takes the following values:

x	0	1	2	3
$p(x)$	2	1	0	1

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2

9. The domain of the function $g(x) = \max\{\sin x, \cos x\}$ is $(-\infty, \infty)$. Find its range.

(A) $(-1, 1]$ (B) $[0, 1]$ (C) $\left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

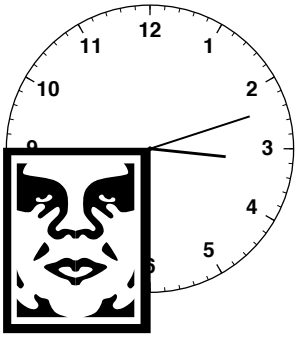
(D) $\left[\frac{-1}{\sqrt{2}}, 1\right]$ (E) $\{1\}$

10. If x and y are real numbers such that $x + y = 1$ and $xy = -1$, which of the following statements is true?

(A) $x^2 + y^2 = 2$ (B) $x^3 + y^3 = 3$ (C) $x^4 + y^4 = 6.$

(D) $x^5 + y^5 = 11$ (E) $x^6 + y^6 = 17$

11. A sticker covers one quarter of the face of a clock. For what fraction of the day is it not possible to see both hands?



(A) $\frac{3}{16}$ (B) $\frac{4}{16}$ (C) $\frac{5}{16}$ (D) $\frac{7}{16}$ (E) $\frac{9}{16}$

12. Let x and y be nonzero real numbers for which

$$x + \frac{6}{x} = 2y + \frac{3}{y}.$$

If $x/y \neq 2$, find the product xy.

(A) -3 (B) -1 (C) 1 (D) 3 (E) $\sqrt{5}$

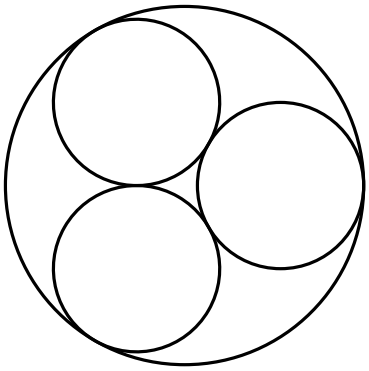
13. For which values of k will

$$\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta} = k$$

have exactly 2 solutions on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) $k \geq 0$ (B) $k \geq 2$ (C) $k > 2$ (D) $k \geq 4$ (E) $k > 4$

14. Three circles of radius r are tangent to one another and are circumscribed by a circle of radius R. Which statement is true?



(A) $r = \frac{R}{3}$ (B) $r = \frac{R}{2}$

(C) $r = R(2\sqrt{3} - 3)$ (D) $r = R\frac{\sqrt{3}-1}{2}$

(E) None of the above

15. How many real numbers x satisfy $|2x - 3| + |x - 3| = |4x - 1|$?

(A) 1 (B) 2

(C) 3 (D) 4

(E) There are no solutions.

16. $\int_{-2}^{2} (\sin^3 x + |x|)\, dx$ equals

(A) $\frac{1}{4}\cos^4 2 - \frac{1}{4}\cos^4(-2) + 4$ (B) $\frac{1}{4}\sin^4 2 - \frac{1}{4}\sin^4(-2)$

(C) $2\cos 2 + \frac{2}{3}\cos^3 2 + 4$ (D) $\frac{1}{2}\sin^4 2 + 4$

(E) None of the above.

17. Which of the following lines is an asymptote of $y = \sqrt{x^2 + x}$?

(A) $y = x - 2$ (B) $y = -x + 1/2$

(C) $y = -x + 2$ (D) $y = x + 2$

(E) $y = x + 1/2$

18. $\lfloor x \rfloor$ denotes the greatest integer which is less than or equal to x. For example, $\lfloor 6.25 \rfloor = 6$, $\lfloor 4 \rfloor = 4$, and $\lfloor \pi \rfloor = 3$. What can you say about the integer solutions n of the following equation?

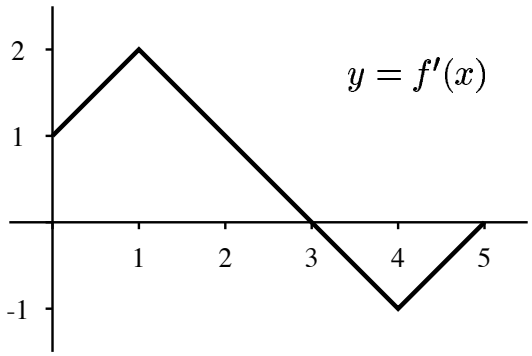
$$\lfloor \sqrt[4]{1} \rfloor + \lfloor \sqrt[4]{2} \rfloor + \lfloor \sqrt[4]{3} \rfloor + \cdots + \lfloor \sqrt[4]{n} \rfloor = 2n$$

(A) There are no solutions. (B) There is more than one solution.

(C) The only solution is $n = 85$ (D) The only solution is $n = 90$

(E) The only solution is $n = 95$

19. The graph of $f'(x)$ appears at right. If $f(0) = 2$, what is $f(5)$?



(A) -1.5 (B) 0 (C) 2.5 (D) 4.5 (E) 5.5

20. The line tangent to $y = x^4 - 2x^2 + 4x + 1$ at $x = -1$ is also tangent to the curve at what other x-value(s)?

(A) At $x = 0$ only. (B) At $x = 1$ only.

(C) At $x = 2$ only. (D) At both $x = 0$ and $x = 1$.

(E) At both $x = 0$ and $x = 2$.

21. The bottom row of a brick wall contains 49 bricks. The next row up contains 47, and each subsequent row contains 2 fewers bricks than the row immediately below it. The number of bricks in the top row is 3. If the wall is one brick thick, what is the total number of bricks in the wall?

(A) 575 (B) 576 (C) 624 (D) 625 (E) 650

22. Find the constant term in the expansion

$$\left(x - \frac{1}{x}\right)^6 = x^6 + \cdots + \frac{1}{x^6}$$

(A) 20 (B) -20 (C) 16 (D) -15 (E) -15

23. $\left(\cos\frac{3\pi}{10} + i\sin\frac{3\pi}{10}\right)^{10}$ equals

(A) 1 (B) -1 (C) i (D) $-i$ (E) $\frac{1+i}{\sqrt{2}}$

24. Which of these polynomials is **not** a factor of $x^{10} - 1$?

(A) $x^5 - 1$

(B) $x^9 + x^8 + x^7 + \cdots + x + 1$

(C) $x^9 - x^8 + x^7 - \cdots + x - 1$

(D) $x^8 + x^6 + x^4 + x^2 + 1$

(E) $x^8 - x^6 + x^4 - x^2 + 1$

25. A club of 15 members must chose a delegation of 4 members to serve at a convention. Ewa and Bob each refuse to serve in the delegation unless the other one also serves. How many delegations are possible?

(A) 793 (B) 806

(C) 819 (D) 832

(E) None of the above

Answers

1. a
2. d
3. c
4. b
5. e
6. b
7. d
8. b
9. d
10. d
11. d
12. d
13. e
14. c
15. b
16. a
17. e
18. e
19. d
20. b
21. c
22. b
23. b
24. e
25. a