

College of Charleston
Math Meet 2006
Written Test – Level 3

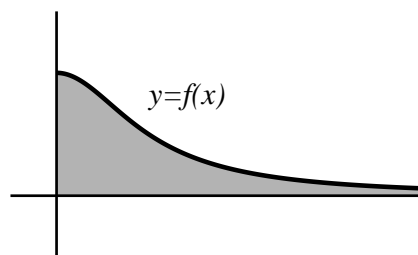
- Find the number of distinct real solutions to $4x^8 + 3x^6 - 7x^2 = 1$.
 (A) 8 (B) 6 (C) 4 (D) 2 (E) 0
- Find the period of the function $\sin(2x) \sin(6x) \sin(8x)$.
 (A) $\pi/3$ (B) $\pi/4$ (C) $\pi/2$ (D) π (E) 2π
- The number of solutions x to the equation

$$|x| + |x - 1| + |x - 2| + \cdots + |x - 99| = m$$

depends on m . For example, there are no solutions if $m = -1$. By choosing m correctly, what's the greatest number of solutions possible?

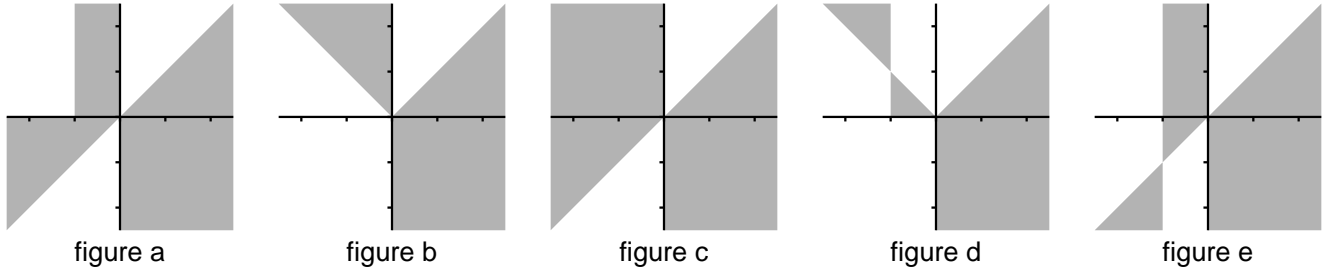
- (A) 1
(B) 2
(C) 3
(D) 4
(E) For the right m , there are infinitely many solutions.
- When $x^{100} - 1$ is divided by $x - 2$, the constant term in the quotient is
 (A) 2^{99} (B) $2^{100} - 1$ (C) 1
 (D) -1 (E) 0
- What is the range of $f(x) = (1 + \sec^{-1} x)(1 + \cos^{-1} x)$?
 (A) $(-\infty, \infty)$ (B) $[1, \pi^2]$ (C) $\{1, \pi^2\}$
 (D) $[0, (1 + \pi)^2]$ (E) $\{1, (1 + \pi)^2\}$

- $f(x)$ is the nonnegative function defined on $[0, \infty)$ whose graph is shown in the figure. The shaded area trapped between the graph of $f(x)$ and the x -axis is 4. What's the area trapped between the x -axis and the graph of $2f(2x)$?



- (A) 16 (B) 8 (C) 4 (D) 2 (E) 1
- The lengths of the sides of a triangle are consecutive integers, and the largest angle is twice the smallest angle. The cosine of the smallest angle must equal
 (A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{9}{14}$ (D) $\frac{7}{10}$ (E) $\frac{8}{13}$

8. Find the graph of the inequality $\frac{y}{x} < \frac{y+1}{x+1}$. (Assume, as usual, that the x -axis is horizontal and the y -axis is vertical.)

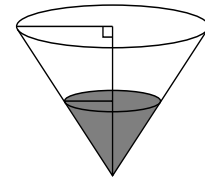


- (A) figure a (B) figure b (C) figure c (D) figure d (E) figure e

9. If $p(x)$ is a polynomial, and if $p(x) + ip(1-x) = \sqrt{2}x^5$, what's the coefficient of x^5 when $p(x)$ is written in the standard power form $p(x) = a + bx + cx^2 + \dots$? (Here, i denotes $\sqrt{-1}$.)

- (A) $1 - i$ (B) $\sqrt{2}(1 - i)$ (C) $\sqrt{2}(1 + i)$
 (D) $\frac{1 - i}{\sqrt{2}}$ (E) $\frac{1 + i}{\sqrt{2}}$

10. A tank in the shape of an inverted cone is partially filled with water. When more water is added, the height of the water in the tank increases by 10 cm, and the volume of water in the tank doubles. Find the original height (in cm) of water in the tank.



- (A) $\frac{10}{\sqrt[3]{2} - 1}$ (B) $\frac{10}{\sqrt[3]{2}}$
 (C) $5\sqrt[3]{4}$ (D) $\frac{5}{\sqrt[3]{2} + 1}$
 (E) Not enough information.

11. A coin is to be tossed repeatedly until heads occurs on 2 consecutive tosses. What's the probability that at least 5 tosses will be necessary?

- (A) $1/2$ (B) $3/16$ (C) $3/32$ (D) $5/32$ (E) $5/64$

12. If $f(x) = \frac{2x+3}{5x-7}$, then $f^{-1}(x) =$

- (A) $\frac{3x+5}{7x-2}$ (B) $\frac{2x-7}{5x+3}$ (C) $\frac{5x+2}{3x-7}$
 (D) $\frac{7x+3}{5x-2}$ (E) $\frac{5x-7}{2x+3}$

13. Everyone in the city of Qs drinks either coffee or tea, but not both. Every day, 10% of the coffee drinkers switch to tea, and 30% of the tea drinkers switch to coffee, and yet the total number of coffee drinkers remains the same, and the total number of tea drinkers remains the same. What is the ratio of coffee drinkers to tea drinkers?
 (A) 2:1 (B) 3:1 (C) 3:2 (D) 1:1 (E) 4:3
14. Find the union of all intervals of the form $\left[1 + \frac{1}{n}, n + \frac{2}{n}\right]$ where n is a positive integer.
 (A) $(0, \infty)$ (B) $[0, \infty)$ (C) $(2, 3)$ (D) $(1, \infty)$ (E) $[2, 3]$
15. The function $f(x) = \sec x - \tan x$ is undefined at $x = \frac{\pi}{2}$. Which best describes the graph of $f(x)$ near $x = \frac{\pi}{2}$? ("VA" stands for "Vertical Asymptote.")
 (A) VA at $x = \frac{\pi}{2}$. y approaches $+\infty$ as x nears $\frac{\pi}{2}$.
 (B) VA at $x = \frac{\pi}{2}$. y approaches $-\infty$ as x nears $\frac{\pi}{2}$.
 (C) No VA at $x = \frac{\pi}{2}$; instead graph has a hole at height 0 there.
 (D) No VA at $x = \frac{\pi}{2}$; instead graph has a hole at height 1 there.
 (E) No VA at $x = \frac{\pi}{2}$; instead graph has a hole at height -1 there.
16. Solve for x in the equation $e^{e^x} = 2$.
 (A) $x = \pm \ln 2$ (B) $x = \ln(2)$
 (C) $x = 2 \ln(2)$ (D) $x = \ln(2) \ln(2)$
 (E) $x = \ln(\ln(2))$
17. Find the sum of all solution(s) x to $|2x - 1| - |x - 5| = 3$.
 (A) -4 (B) -1
 (C) 1 (D) 3
 (E) The equation has no solutions.
18. Let $\llbracket x \rrbracket$ denote the greatest integer less than or equal to x . How large must x be to ensure that $\frac{x}{\llbracket x \rrbracket}$ is less than .001 unit away from 1?
 (A) $x \geq 100$ (B) $x \geq 500$ (C) $x \geq 999$
 (D) $x \geq 1000$ (E) $x \geq 1001$.
19. The 12th derivative of $f(x) = e^x \sin x$ is
 (A) $-64e^x \sin x$ (B) $64e^x \cos x$ (C) $64e^x \sin x$
 (D) $-12e^x \sin x$ (E) $12e^x \cos x$

2006 Answers / Level 3 Test

1. D
2. C
3. E
4. A
5. E
6. C
7. B
8. E
9. E
10. A
11. A
12. D
13. B
14. D
15. C
16. E
17. A
18. D
19. A
20. E
21. B
22. B
23. B
24. D
25. C