College of Charleston
Math Meet 2013
Written Test – Level 3

1. Which of these is equal to $\log_{10}\left(\dfrac{10^{2,000,000}+10^{1,999,999}}{11\cdot 10^{1,000,000}}\right)$?

   (A) 1 (B) $999,999$ (C) $1,000,001$ (D) $1,999,999$ (E) $2,000,001$

2. Ben and Zach are playing a game involving a six-sided die. If Ben rolls a 1 or 2, then he loses the game. If Zach rolls a 1, 2, or 3, then Zach loses the game. They take turns rolling the die, beginning with Ben, and continue until someone loses the game.
   Assuming that the six outcomes of the die—1, 2, 3, 4, 5, 6—are equally likely, what's the probability that Ben will win the game?

   (A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{2}{3}$ (E) $\dfrac{4}{7}$

3. The hour hand on a clock is 4 cm long, and the minute hand is 6 cm. How fast (in cm^2/min) is the area of the triangle determined by the hands decreasing at 8:00?

   (A) $\dfrac{11\pi}{360}$ (B) $\dfrac{11\pi}{120}$ (C) $\dfrac{11\pi}{60\sqrt{2}}$ (D) $\dfrac{11\pi\sqrt{3}}{60}$ (E) $\dfrac{11\pi}{60}$

4. What is the amplitude of $3\sin(x)+4\cos(x)$?

   (A) 1 (B) 4 (C) 5 (D) 7 (E) none of these

5. Use the table of values:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	3	4	2	-2
2	-1	-5	3	9
3	2	10	-1	11

   to find $\dfrac{d}{dx}\left[g(f(x))\right]$ at $x=-1$.

   (A) 8 (B) -8 (C) 44 (D) 27 (E) -20

6. A teacher says "When the positive integer x is divided by 5, the remainder is 4. When x is divided by 7, the remainder is 4. When x is divided by 9, the remainder is 7. When x is divided by 11, the remainder is 5."
   A student interrupts, "You mean like x equals 214?"
   The teacher hastily replies, "Yes, except that x is not 214."
   The smallest possible value of x is

   (A) less than 999. (B) between 1000 and 1999. (C) between 2000 and 2999.
   (D) between 3000 and 3999. (E) over 4000.

7. If $g(x) = \sin x$, find $\lim_{h \to 0} \dfrac{g(x-h) - g(x)}{h}$.

(A) $\sin x$ (B) $-\sin x$ (C) $\cos x$

(D) $-\cos x$ (E) The limit does not exist.

8. Find the number of real solutions to the equation $x^5 + 2x^3 + 3 = 0$.

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

9. Let $f(x) = \tan^{-1}\left(\dfrac{e^x - e^{-x}}{2}\right)$. Which of the following is equal to $f'(x)$?

(A) $\dfrac{1}{e^x - e^{-x}}$ (B) $\dfrac{2}{e^x + e^{-x}}$ (C) $\dfrac{e^{2x}+1}{e^{2x}-1}$

(D) $\dfrac{e^x - e^{-x}}{e^{2x} + e^{-2x}}$ (E) none of these

10. What is the acute angle between the (oblique) asymptotes of the hyperbola

$$3x^2 - 7xy + 2y^2 + 3x + 4y + 2 = 0?$$

(A) 30° (B) 45° (C) 60°

(D) 75° (E) none of these

11. Suppose you have a collection of coins from five different countries. Each of those countries has five different denominations of coins, and you have one of each. Your coins are distributed into five boxes so that each box contains coins from exactly two countries and no box contains all of the coins from any of the countries. What is the smallest number of boxes you need to open to be certain to find all five coins from at least one country?

(A) 5 (B) 4 (C) 1 (D) 2 (E) 3

12. What is the area of the triangle with vertices $(1,1)$, $(2,2)$, and $(3,4)$?

(A) $\dfrac{1}{6}$ (B) $\dfrac{1}{\sqrt{3}}$ (C) $\sqrt{2}$

(D) 3 (E) none of these

13. In a Sudoku puzzle, one must place a digit (1,2,. . . ,9) in each of the 1×1 squares so that each row, each column, and each of the 3×3 grids outlined in bold contains all 9 digits. Find the digit that must go in the shaded square.

	8	5			1			4
2	7			4				
			5					
	2	9		1			4	6
4			7	9	5			1
7	1			6		9	5	
					7			
				8			6	3
1			2			5	8	

(A) 3 (B) 5 (C) 6 (D) 8 (E) 9

14. Consider a regular 12-gon $ABCDEFGHIJKL$ of diameter 2. What is the area of the trapezoid $ABCD$?

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{3}}{2}-\frac{1}{2}$ (C) $\frac{3}{4}$

(D) $\frac{\sqrt{3}}{2}$ (E) none of these

15. Let $f(x)=x+\frac{1}{x}$. Which of the following values is smallest?

(A) $f\left(\frac{1}{3}\right)$ (B) $f\left(\frac{1}{2}\right)$ (C) $f\left(\sqrt{3}\right)$ (D) $f(e)$ (E) $f(\pi)$

16. Find the smallest positive solution to $0=\sin x+\sin 2x+\sin 3x+\sin 4x$.

(A) $\frac{3\pi}{8}$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{2}$

(D) π (E) none of these

17. How many positive integers are twice the sum of their digits?

(A) none of them (B) 1 (C) 2

(D) 15 (E) infinitely many

18. Let C be the unit circle centered at the origin. Place a point P on C in the first quadrant. Place a point Q on C in the second quadrant. Place a point R on C in the third quadrant. What is the maximum possible area for the triangle PQR?

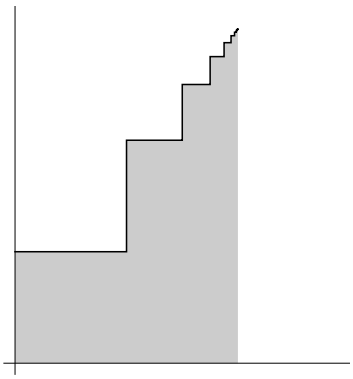
(A) 1 (B) 2 (C) $\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$ (E) none of these

19. Five men, Mr. A, Mr. B, Mr. C, Mr. D, and Mr. E, owned parakeets. Their wives owned cats. Each woman's cat killed one of the five parakeets.
Mrs. A's cat killed the parakeet owned by the man married to the owner of the cat which killed Mr. E's parakeet. Mr. A's parakeet was killed by Mrs. B's cat. Mr. D's parakeet was killed by the cat owned by the woman who married the man whose parakeet was killed by Mrs. C's cat.
Who was the owner of the parakeet killed by Mrs. D's cat?

(A) Mr. A (B) Mr. B (C) Mr. C (D) Mr. D (E) Mr. E

20. Starting at $(0,1)$, draw a horizontal line segment one unit to the right (to $(1,1)$) and from there, draw a vertical line segment one unit up to $(1,2)$. Continue from there drawing a horizontal line segment of length $1/2$ followed by a vertical line segment of length $1/2$, and then a horizontal line segment of length $1/4$ and a vertical line segment of length $1/4$, followed by segments of length $1/8$, etc. What is the area of the region between this path and the x-axis?

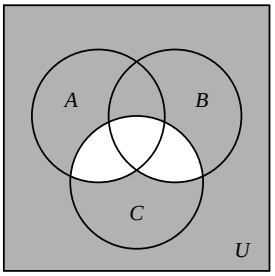


(A) $\frac{10}{3}$ (B) $\frac{17}{5}$ (C) $\frac{7}{2}$

(D) 4 (E) none of these

21. If A, B, and C are subsets of the set U, then
- $A \cap B$ is the set of all elements that belong to both A **and** B,
- $A \cup B$ is the set of all elements that belong to A **or** B (or both), and
- A^c is the set of all elements of U that are **not** in A.

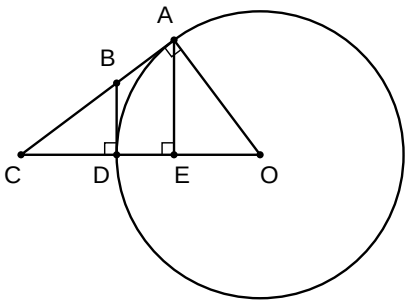
In the diagram, A, B and C are represented by circles and U is represented by a square.
Which of the following expressions describes the region shaded in the diagram?



(A) $(A^c \cap B^c) \cup C^c$ (B) $A^c \cup (B^c \cap C^c)$ (C) $A^c \cap (B^c \cup C^c)$

(D) $(A^c \cup B^c) \cap C^c$ (E) $A^c \cap B^c \cap C^c$

22. O is the center of a circle of radius 3 cm. A and D are points on the circumference of that circle. Points ABC and points CDEO are colinear. If AC has length 4 cm, find the area (in cm^2) of the trapezoid ABDE.



(A) 99/100 (B) 99/50 (C) 117/50 (D) 228/75 (E) 67/50

23. The function $\dfrac{1}{1+e^x}$ is the sum of an even function $f(x)$ and an odd function $g(x)$. Find $f(x)-g(x)$.

(A) $\dfrac{1}{1-e^x}$ (B) $\dfrac{1}{e^x-1}$ (C) $\dfrac{1}{e^{-x}+1}$ (D) $\dfrac{-1}{1+e^x}$ (E) $\dfrac{1}{1-e^{-x}}$

24. How many real solutions are there to the equation

$$x^8-x^7+4x^6-3x^5+6x^4-3x^3+4x^2-x+1=0 \quad ?$$

(A) 0 (B) 2 (C) 3 (D) 4 (E) more than 4

25. Find $f(x)$ if $f(1)=6$ and $f(x+y)-f(x)=y$ for all real numbers x and y.

(A) $f(x)=8-2x$ (B) $f(x)=7-x$ (C) $f(x)=6$

(D) $f(x)=5+x$ (E) $f(x)=4+2x$

2013 Answers / Level 3 Test

1. B
2. A
3. E
4. C
5. C
6. D
7. D
8. E
9. B
10. B
11. E
12. E
13. D
14. A
15. C
16. B
17. B
18. E
19. B
20. A
21. A
22. C
23. C
24. A
25. D