College of Charleston
Math Meet 2015
Written Test – Level 3

1. Let x_1 and x_2 be the two solutions to the equation

$$x^2 - x - 3 = 0.$$

Let $X_i = x_i \times e^{x_i}$ where $i = 1$ or $i = 2$ and where $e \approx 2.718$ is the base of the natural logarithm. Which of these integers is closest to the product $X_1 \times X_2$?

(A) -9 (B) -8 (C) -6 (D) -5 (E) -3

2. L and K are perpendicular lines in the xy-plane. The three lines L, K, and $y = 4 - x$ intersect at a single point. L's y-intercept is 7, and K's x-intercept is 1. Find the sum of the slopes of L and K.

(A) 0 (B) 2/3 (C) -3/2 (D) -2/3 (E) 3/2

3. A coin is biased so that the probabilities of heads and tails are $2/3$ and $1/3$ respectively. A second coin is biased so that the probabilities of heads and tails are $3/5$ and $2/5$ respectively. If both coins are tossed, find the probability of at least one head.

(A) $\frac{13}{15}$ (B) $\frac{2}{5}$ (C) $\frac{8}{15}$

(D) $\frac{3}{5}$ (E) None of the above

4. Let $f(x) = \sin\left(5x^2\right) - 5\sin\left(x^2\right)$. The first several derivatives evaluated at $x = 0$ are all 0. For example, $f'(0) = 0$ and $f''(0)$ and $f^{(3)}(0) = 0$. What is the smallest value of k for which $f^{(k)}(0) \neq 0$?

(A) 4 (B) 5 (C) 6

(D) 10 (E) none of these

5. Professor Calini always tells the truth, except on the weekend (Saturday and Sunday), when she lies all the time. On how many days of the week can Professor Calini say "*If I did not tell the truth yesterday, then I will not lie tomorrow*?"

(A) 0 (B) 1 (C) 3 (D) 5 (E) 7

6. For which real values of c does the equation $x^4 + x^2 + c = 0$ have at least one real solution?

(A) There are no such values of c (B) $c \leq \frac{1}{4}$

(C) $-1 \leq c \leq 1$ (D) $0 \leq c$

(E) $c \leq 0$

7. Which of the given functions satisfies

$$f(n+1) - f(n) - 6f(n-1) = 0$$

for all integers n?

(A) $f(n) = 3 \cdot 2^n - 2 \cdot 3^n$ (B) $f(n) = 3 \cdot 2^n + 2 \cdot (-3)^n$

(C) $f(n) = 1 - 2 \cdot 3^n$ (D) $f(n) = 3 \cdot (-2)^n + 2 \cdot (-3)^n$

(E) $f(n) = 3 \cdot (-2)^n - 2 \cdot 3^n$

8. Simplify $\sin(x) + \cos(x) + \sec(x) + \csc(x)$.

(A) 1

(B) $(\csc(x) + \sec(x))(1 + \sin(x)\cos(x))$

(C) $\cot(x) + \tan(x)$

(D) $\dfrac{\csc(x)\sec(x)}{\cot(x) + \tan(x)}$

(E) none of these

9. Let C be the circle $x^2 + y^2 = 27$. No point on C has integer coordinates (meaning x and y can't both be integers). What is the distance from C to the nearest point with integer coordinates?

(A) $\dfrac{1}{\sqrt{27}}$ (B) $\dfrac{1}{3}$ (C) $\sqrt{3} - \sqrt{2}$

(D) $\sqrt{27} - \sqrt{26}$ (E) none of these

10. Let $a_0 = 3$ and $a_n = \dfrac{2}{3}a_{n-1} + 7$. What is $\lim_{n\to\infty} a_n$?

(A) 19 (B) 21 (C) $\dfrac{70}{3}$

(D) ∞ (E) none of these

11. How many different numbers can be written in the form $x + y + z$ where x, y, and z are in $\{1, 3, 5, \ldots, 21\}$?

(A) 9261 (B) 4830 (C) 441 (D) 231 (E) 31

12. If $f(x)$ is positive and decreasing on the interval $(0, \infty)$, which of the following statements must be true?

(A) $f(x)$ is continuous on $(0, \infty)$. (B) $\lim_{x\to\infty} f(x) = -\infty$.

(C) $\lim_{x\to\infty} f(x) > 0$. (D) $\lim_{x\to\infty} f(x) = 0$.

(E) none of the above.

13. What is the derivative of $x|x|$?

(A) $x|x|$ isn't differentiable.

(B) $2x$

(C) $x + |x|$

(D) $2|x|$

(E) $x|x|$ is differentiable but its derivative is none of these.

14. Which of the following numbers is a solution to $15\sin x + 12\tan x = 28$?

(A) $\tan^{-1}\dfrac{4}{3}$ (B) $\sin^{-1}\dfrac{15}{17}$ (C) $\tan^{-1}\dfrac{12}{5}$ (D) $\tan^{-1}\dfrac{24}{7}$ (E) $\sin^{-1}\dfrac{3}{7}$

15. Which of the following numbers is largest?

(A) $\ln 3$ (B) $\ln\ln 3$ (C) $\dfrac{1}{\ln 3}$ (D) $3 - \ln 3$ (E) $\dfrac{3}{\ln 3}$

16. Which of these is a valid antiderivative for $(x^2-1)e^x$?

(A) $(\frac{1}{3}x^3 - x + 1)e^x$ (B) $(\frac{1}{3}x^3 - x)e^x$ (C) $(x^2 + 2x + 1)e^x$

(D) $(x-1)^2 e^x$ (E) none of these

17. Let $ABCDEFGHIJKL$ be a regular 12-gon. Consider the following angles: $\angle CAB$, $\angle DAC$, $\angle EAD$, $\angle FAE$. Which one of the following statements is true about those four angles?

(A) $\angle CAB$ is larger than the rest (B) $\angle DAC$ is larger than the rest

(C) $\angle EAD$ is larger than the rest (D) $\angle FAE$ is larger than the rest

(E) all four angles all the same size

18. $(\log_2 4)(\log_3 5)(\log_4 6)(\log_5 7)(\log_6 8)(\log_7 9) =$

(A) 6 (B) 5 (C) 4

(D) 3 (E) none of these

19. Which of the following expressions is equal to $\dfrac{1}{x} - \dfrac{1}{x^3} + \dfrac{1}{x^5} - \dfrac{1}{x^7} + \dfrac{1}{x^9} - \dfrac{1}{x^{11}}$?

(A) $\dfrac{1}{x^{36}}$ (B) $\dfrac{x^{12}-1}{x^{13}+x^{11}}$ (C) $\dfrac{x^{12}-x}{x^{11}}$

(D) $\dfrac{x^{10}-1}{x-1}$ (E) none of these

20. If speed is measured in miles per minute, then "pace," measured in minutes per mile, is the reciprocal of speed. Yesterday, Ewa started running at a pace of 11 minutes per mile. After one mile, she was running 10.5 minutes per mile. If her pace was a linear function of the distance she had traveled, how long did it take her to run 3 miles?

(A) 31 minutes. (B) 30 minutes and 45 seconds

(C) 30 minutes and 30 seconds (D) 30 minutes and 15 seconds

(E) 30 minutes

21. How many values of θ in $[0, \pi]$ satisfy $\cos 2\theta \cos\theta = 1 + \sin\theta \sin 2\theta$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

22. $ABCD$ is a square, and E is a point in the interior of the square. If $\overline{AE} = 1$, $\overline{BE} = 2$, and $\overline{CE} = 3$, find $\overline{DE}$.

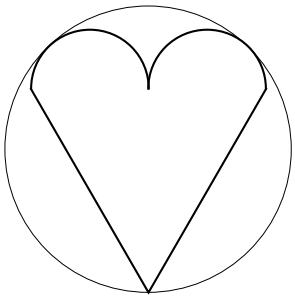
(A) $\sqrt{4}$ (B) $\sqrt{6}$ (C) $\sqrt{10}$ (D) $\sqrt{12}$ (E) $\sqrt{13}$

23. Suppose that the College of Charleston always holds the Math Meet on the second Saturday of February. This year, that happens to be February 14^{th} (Valentine's Day). Which year would be the next time that the Math Meet is on Valentine's Day?

(A) 2020 (B) 2022 (C) 2026

(D) 2043 (E) none of these

24. Take an equilateral triangle of side length 2. Pick one side of the triangle and attach two semicircles of diameter 1. What is the radius of the smallest circle that can contain this heart-shaped figure?



(A) $\dfrac{3}{2\sqrt{3}-1}$ (B) 1 (C) $\dfrac{2}{\sqrt{3}}$

(D) $\sqrt{3}$ (E) none of these

25. If $2\log(x-2y) = \log x + \log y$, find x/y. Assume that x, y, and $x-2y$ are strictly greater than zero.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

2015 Answers / Level 3 Test

1. B
2. C
3. A
4. C
5. D
6. E
7. E
8. B
9. D
10. B
11. E
12. E
13. D
14. A
15. E
16. D
17. E
18. A
19. B
20. B
21. C
22. B
23. C
24. A
25. D