College of Charleston
Math Meet 2019
Written Test – Level 3

1. Find the solution set: $|2x+1| + |x-4| \leq |x-6|$

   (A) $\left[-\frac{3}{2}, \frac{9}{4}\right]$ (B) $\left[-\frac{3}{2}, \frac{1}{2}\right]$ (C) $\left[\frac{1}{2}, \frac{9}{4}\right]$ (D) $\left[-\frac{1}{2}, \frac{9}{4}\right]$ (E) $\left[-\frac{9}{4}, \frac{3}{2}\right]$

2. On the interval $(-1, 1)$, the function $\dfrac{x-1}{x^3+1}$ can be written as a sum $f(x) + g(x)$ where f is even and g is odd. Find $f(x) - g(x)$.

   (A) $\dfrac{1}{x^2-x+1}$ (B) $\dfrac{1}{x^2+x+1}$ (C) $\dfrac{-1}{x^2+x+1}$

   (D) $\dfrac{1+x}{x^3-1}$ (E) $\dfrac{1-x}{x^3+1}$

3. Oman and Bar are playing a game. Each rolls a fair 6-sided die. (This means that each player receives a number in $\{1, 2, 3, 4, 5, 6\}$ each with equal probability.) If Oman rolls a 6 and Bar rolls a number less than 5, then Oman wins. If Oman rolls a number less than 6 and Bar rolls a 5 or 6, then Bar wins. In case of any other combination of rolls, the players reroll until someone wins. Find the probability that Oman will win.

   (A) 1/6 (B) 2/7 (C) 3/11 (D) 4/36 (E) 5/49

4. Find the third derivative of $f(x) = \dfrac{4x^3 - 8x^2 + 4x - 3}{(x-1)^2}$ at $x = 0$.

   (A) -52 (B) 62 (C) -72 (D) 82 (E) -92

5. Which of these is always equal to $2\sin(x)\cos(x)$?

   (A) $\cos(2x)$

   (B) $\sin(x+\frac{\pi}{5})\cos(x-\frac{\pi}{5}) - \sin(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (C) $\sin(x+\frac{\pi}{5})\cos(x-\frac{\pi}{5}) + \sin(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (D) $\sin(x+\frac{\pi}{5})\sin(x-\frac{\pi}{5}) - \cos(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

   (E) $\sin(x+\frac{\pi}{5})\sin(x-\frac{\pi}{5}) + \cos(x-\frac{\pi}{5})\cos(x+\frac{\pi}{5})$

6. A sequence of numbers $a_0, a_1, a_2, \ldots$ satisfies the **recurrence relation**

   $$a_{n+2} = a_n \text{ for all } n \geq 0.$$

   Another sequence $b_0, b_1, b_2, \ldots$ satisfies

   $$b_{n+1} = 2b_n \text{ for all } n \geq 0.$$

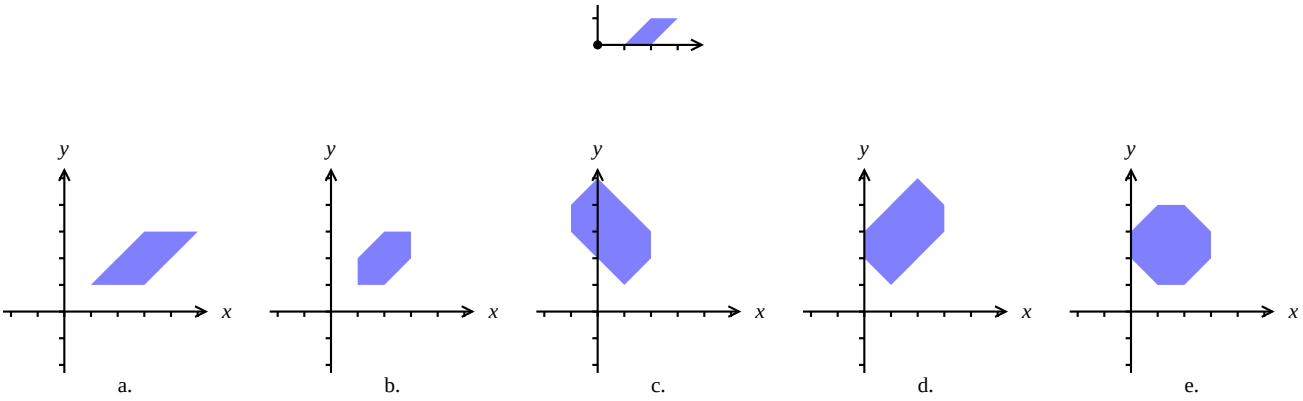
   Let $c_n = a_n + b_n$ for all $n \geq 0$. Which of these recurrence relations must be true (for all $n \geq 0$)?

   (A) $c_{n+2} = 2c_{n+1} + c_n$ (B) $c_{n+2} = c_{n+1} - 2c_n$

   (C) $c_{n+2} = 5c_n$ (D) $c_{n+3} = 2c_{n+2} + c_{n+1} - 2c_n$

   (E) $c_{n+3} = 2c_{n+2} - c_{n+1} + 2c_n$

7. When Rohn tries to throw a football, his aim is poor but very predictable. See the top figure below. If he stands on the dot ● and throws in the direction of the arrow, the ball will always land in the shaded parallelogram.
Suppose Rohn stands at the origin and, aiming in the positive horizontal direction, throws a ball. Then he walks to where the ball landed, aims in the positive vertical direction, and throws the ball again. Which figure shows the shaded region in which the ball could land the second time?



(A) Figure a. (B) Figure b. (C) Figure c. (D) Figure d. (E) Figure e.

8. You are standing at the point $(-4, 0)$ on the shore of a pond occupying the region $\{(x, y) : y > (x + 1)(x + 4)\}$. A friend is standing at the point $(4, 0)$ on the shore of a second pond occupying the region $\{(x, y) : y < (1 - x)(x - 4)\}$. You wish to walk to your friend along the shortest possible route, and you don't want to get your feet wet. Which of the following points will you cross on your path?

(A) $(2, 2)$ (B) $(-1/2, 7/4)$ (C) $(1, 0)$

(D) $(0, 1)$ (E) $(\sqrt{2}, 5\sqrt{2} - 6)$

9. What is the smallest value of the function x^x for $\frac{1}{4} \leq x \leq 1$?

(A) 1 (B) $e^{-1/e}$ (C) $\frac{1}{\sqrt[3]{3}}$

(D) $\frac{1}{\sqrt[4]{4}}$ (E) none of these

10. Which of these is a simpler way to write the number $\frac{\frac{1}{c-1} + 1}{\frac{1}{c} - \frac{1}{c-1}}$?

(A) $-2c$ (B) $2c$ (C) $c - c^2$ (D) $c^2 - c$ (E) $-c^2$

11. Let l be the tangent line to the circle $x^2 + y^2 = 169$ at the point $(5, -12)$. Find the x-intercept of l.

(A) $\left(\frac{144}{3}, 0\right)$ (B) $\left(\frac{169}{5}, 0\right)$ (C) $\left(\frac{156}{7}, 0\right)$ (D) $\left(\frac{126}{5}, 0\right)$ (E) $\left(\frac{145}{8}, 0\right)$

12. The keys in a square keypad of side 4 are numbered 1 through 16. Secret agent 007 must press two keys simultaneously in order to access the enemy headquarters. If he presses the wrong two keys, an alarm will sound. The only piece of information he knows is that the two keys are not adjacent (i.e., they have no side or vertex in common). What is the probability that he will guess the right combination and gain access to the enemy headquarters?

(A) $\frac{1}{64}$ (B) $\frac{1}{78}$ (C) $\frac{1}{128}$ (D) $\frac{1}{156}$ (E) $\frac{1}{160}$

13. A factory assembly line produces a certain item, and on average 10% of the items are defective. Suppose a random sample of 10 items is taken. What is the probability that exactly 3 items in the sample are defective?

(A) $\frac{10!}{7!\,3!}(0.9)^7(0.1)^3$ (B) $(0.9)^7(0.1)^3$ (C) $\frac{10!}{7!}(0.9)^7(0.1)^3$

(D) $\frac{10!}{7!\,3!}(0.9)^3(0.1)^7$ (E) $(0.9)^3(0.1)^7$

14. Evaluate $(\log_2 3)\,(\log_3 4)$.

(A) 2 (B) 6 (C) $\log_2 6$

(D) $\log_6 12$ (E) none of these

15. Consider the set $S = \{1, 2, \ldots, 2004\}$. How many subsets of S are there such that the sum of their elements equals $2,009,000$?

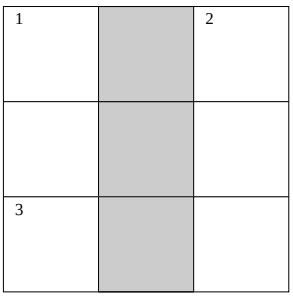
(A) 8 (B) 10 (C) 16 (D) 1002 (E) 2^{2000}

16. The hour hand on a clock is 6 cm long. If the tips of the minute hand and the hour hand are 10 cm apart at 3 o'clock, how far apart (in centimeters) are they an hour later?

(A) $2\sqrt{13}$ (B) $2\sqrt{37}$ (C) $10 + 2\sqrt{12}$

(D) $\sqrt{100 - 48\sqrt{3}}$ (E) $\sqrt{100 + 48\sqrt{3}}$

17. Here is a crossword for you, but each entry is a digit instead of a letter. Each digit between 1 and 9 appears in the crossword once and only once. You are given the following clues:



Across:

1. A square.
3. A number the sum of whose digits is 9.

Down:

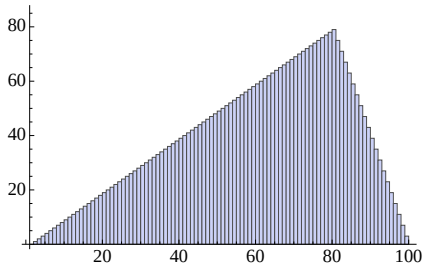
1. A cube.
2. A number with only even digits.

What is the number in the gray column, read from top to bottom?

(A) 820 (B) 937 (C) 764

(D) 793 (E) None of the above

18. A histogram of exam scores is shown. Which of the following is closest to the average exam score?



(A) 50 (B) 60 (C) 70 (D) 75 (E) 80

19. What is the acute angle between the (oblique) asymptotes of the hyperbola

$$3x^2 - 7xy + 2y^2 + 3x + 4y + 2 = 0?$$

(A) 30° (B) 45° (C) 60°
(D) 75° (E) none of these

20. If you completely distribute the multiplication in the expression

$$(a + b + c)^6$$

so that it is a sum of distinct monomials, what is the coefficient of the term a^2bc^3?

(A) 20 (B) $\frac{19}{3}$ (C) 36 (D) $\frac{76}{3}$ (E) 60

21. Which of these is equal to $\log_{10}\left(\frac{10^{2,000,000} + 10^{1,999,999}}{11 \cdot 10^{1,000,000}}\right)$?

(A) 1 (B) $999,999$ (C) $1,000,001$ (D) $1,999,999$ (E) $2,000,001$

22. If $\frac{a + 13b}{3a - b}$ equals 3, then $\frac{a^3}{b^3}$ equals

(A) 64 (B) 27 (C) 8 (D) $\frac{127}{64}$ (E) 1

23. Five candidates—Aay, Bee, Cee, Dea, and Ewa—compete for the Wojcicka Medal in mathematics at the College of Charleston. Each candidate earns between 1 and 5 points in each of five areas—algebra, analysis, topology, discrete math, and logic—and the candidate with the most points wins the medal. There were no ties in any subject, and no ties in overall point totals. Aay came in first with 24 points, Bee came in second, Cee third, Dea fourth, and Ewa fifth. Ewa earned five points in analysis and three points in algebra.
How many points did Cee earn altogether?

(A) 11 (B) 12 (C) 13 (D) 14 (E) 15

24. An interior point of an equilateral triangle is at distances 5, 7 and 8 from the three sides of the triangle. What is the common length of the sides of the triangle?

(A) It cannot be determined
(B) The given configuration cannot exist
(C) 20
(D) $14\sqrt{3}$
(E) $\frac{40}{3}\sqrt{3}$

25. The light rays from a spotlight spread out to form a solid right circular cone, with the light rays making an angle of up to 30 degrees with the centerline of the beam. If the spotlight is 10 feet above the floor, and the centerline of the beam hits the floor at a 60-degree angle of inclination, what is the area (in square feet) that the spotlight covers on the floor?

(A) $100\pi\sqrt{3}$ (B) $25\pi\sqrt{6}$ (C) $\frac{100\pi}{\sqrt{3}}$ (D) $25\pi\sqrt{7}$ (E) $\frac{400\pi}{9}$

2019 Answers / Level 3 Test

1. B
2. D
3. B
4. C
5. C
6. D
7. E
8. A
9. B
10. E
11. B
12. B
13. A
14. A
15. B
16. B
17. D
18. B
19. B
20. E
21. B
22. C
23. C
24. E
25. B