

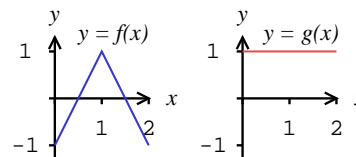
# College of Charleston Math Meet 2022 Written Test – Level 3

1. The surface in the figure was cut from a right circular cylinder of radius one. One cut was made perpendicular to the axis of the cylinder, and the other cut was made at a  $45^\circ$  angle to the axis. The two cuts meet at a single point. Find the area of this surface.



- (A)  $\pi$                       (B)  $\frac{5}{4}\pi$                       (C)  $\frac{4}{3}\pi$                       (D)  $\frac{3}{2}\pi$                       (E)  $2\pi$

2. Let  $f(x)$  be the piecewise-linear function shown in the figure, and extend  $f(x)$  to  $(-\infty, \infty)$  so as to have period 2. Part of the graph of a function



$$g(x) = af(x) + bf\left(\frac{1}{2}x\right) + cf\left(\frac{1}{4}x\right) + df\left(\frac{1}{8}x\right)$$

appears in the figure.

Find  $\int_0^8 g(x) dx$ .

- (A)  $-1/2$                       (B) 0                      (C)  $1/2$   
(D)  $3/2$                       (E) Not enough information.
3. Suppose that  $p(x)$  and  $q(x)$  are polynomials and that  $r(x) = p(x)/q(x)$  satisfies  $\lim_{x \rightarrow 0} r(x) = \lim_{x \rightarrow 3^+} r(x) = \lim_{x \rightarrow \pm\infty} r(x) = \infty$ . What's the smallest possible degree of  $p$ ?
- (A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6

4. I calculated  $\frac{-2 - x + x^2}{1 + x} = -2 + x$  by performing the division with this ascending ordering of terms:

$$\begin{array}{r}
 -2 + x \\
 1 + x \overline{) -2 - x + x^2} \\
 \underline{-( -2 - 2x)} \\
 x + x^2 \\
 \underline{-( x + x^2)} \\
 0
 \end{array}$$

In the same way, I found

$$\frac{p(x)}{q(x)} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + \dots$$

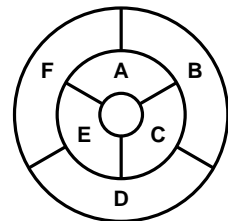
If  $q(1) = 2$ , find  $p(1)$ .

- (A) 0                      (B)  $1/8$                       (C)  $1/4$                       (D)  $1/2$                       (E) 1
5. Which of the following is **not** equivalent to the statement “It rhymes when it runs and rains.”?
- (A) It rhymes or doesn’t run or doesn’t rain.  
 (B) If it rains but not rhymes, then it doesn’t run.  
 (C) When it runs but not rhymes, it doesn’t rain.  
 (D) If it runs, then it rhymes or doesn’t rain.  
 (E) When it rains but not runs, it doesn’t rhyme.
6. Five different numbers are chosen at random from

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}.$$

If any set of five numbers has the same probability of being selected, what’s the probability that their median is 3?

- (A)  $5/28$                       (B)  $1/63$                       (C)  $1/3$                       (D)  $8/27$                       (E)  $1/72$
7. I’ve drawn a map for a role-playing game and wish to color the six countries **A**, **B**, ..., **F** so that each country is colored with one color, and no neighboring countries are colored the same color. If I have four colors available but wish to use the fewest number of colors possible, how many ways are there for me to color my map?



- (A) 256                      (B) 243                      (C) 192                      (D) 24                      (E) 6
8. If the graph of  $f(x)$  has length 3, which of these functions must have a graph of length 6?
- (A)  $2f(x)$                       (B)  $f(2x)$                       (C)  $2f\left(\frac{1}{2}x\right)$   
 (D)  $f\left(\frac{1}{2}x\right)$                       (E)  $\sqrt{2}f\left(\frac{1}{\sqrt{2}}x\right)$

9. I have an original idea for a game, in which the player must guess a passcode consisting of 5 distinct digits (0,1,2,..., 9). After each 5-digit guess, the game will give the player some clues as to which digits in her guess appear in the correct answer. See figure. If a player makes the two guesses shown at the bottom of the figure, what is the remainder of the correct answer when it is divided by 9?

**2** A digit on a WHITE background appears in this place in the correct answer.

**1** A digit on a GRAY background appears in the correct answer, but not in this place.

**0** A digit on a BLACK background does not appear in the correct answer.

<b>6</b>	<b>9</b>	<b>1</b>	<b>8</b>	<b>7</b>
<b>4</b>	<b>1</b>	<b>0</b>	<b>2</b>	<b>5</b>

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5
10. Find the number of passcodes of length 4 that use only the characters A, b, <, and 4 if
- only b or < can follow A,
  - only A, <, or 4 can follow b,
  - only b or 4 can follow <, and
  - only < can follow 4.
- (A) 31                      (B) 28                      (C) 23                      (D) 16                      (E) 11
11. Suppose  $f(x)$  is continuous on  $[0, 1]$ ,  $f(0) = 0$ , and

$$f'(x) = \begin{cases} 1 & \text{if } \frac{1}{2} < x < 1, \\ -1 & \text{if } \frac{1}{4} < x < \frac{1}{2}, \\ 1 & \text{if } \frac{1}{8} < x < \frac{1}{4}, \\ -1 & \text{if } \frac{1}{16} < x < \frac{1}{8}, \\ \vdots & \end{cases}$$

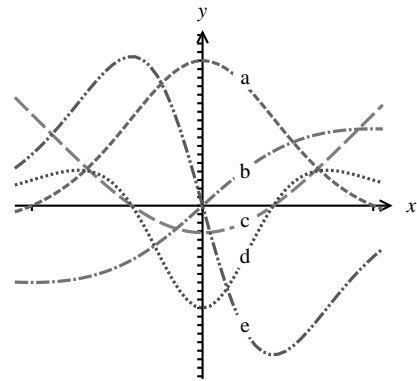
What is the fourth digit to the right of the decimal point in the decimal representation of  $f(1)$ ?

- (A) 3                      (B) 4                      (C) 5                      (D) 6                      (E) 7
12. A sample of bacteria in a Petri dish (labeled “X”) grows exponentially, meaning that its population at time  $t$  equals  $C \cdot a^t$  for some constants  $a$  and  $C$ . Bacteria in a second dish (labeled “Y”) also exhibit exponential growth. The doubling time of population Y (that is, the length of time required for the population to double) is twice that of population X. At time 2, population X equals what population Y was at time 0. At time 8, population X is 8 times population Y. At what time were the two populations the same?
- (A)  $\log_3 81$                       (B)  $2^{3/2}$                       (C)  $2^{5/2}$
- (D)  $\frac{\log_2(16)}{\log_2(2\sqrt{2})}$                       (E) Not enough information.
13. How many subsets of  $\{1, 2, \dots, 100\}$  are there so that the sum of their elements is 5042?
- (A) 3                      (B) 4                      (C) 6                      (D) 7                      (E) 10

14. Suppose  $p(x)$  is a polynomial of degree 5, and that  
 when  $p(x)$  is divided by  $(x - 2)(x - 3)$ , the remainder is  $4 - x$ ,  
 when  $p(x)$  is divided by  $(x - 1)(x - 3)$ , the remainder is  $x - 2$ , and  
 when  $p(x)$  is divided by  $(x - 1)(x - 2)$ , the remainder is  $3x - 4$ .  
 Find the remainder when  $p(x)$  is divided by  $(x - 1)(x - 2)(x - 3)$ .

- (A)  $2x^2 - 11x + 16$   
 (B)  $-2x^2 + 9x - 8$   
 (C)  $2x^2 - 7x + 4$   
 (D)  $-2x^2 + 5x - 2$   
 (E) Impossible. There is no such polynomial  $p$ .

15. The accompanying figure shows the graphs of  $f(x)$  and its first four derivatives. Find the graph of  $f(x)$ .



- (A) a                      (B) b                      (C) c                      (D) d                      (E) e

16. Which of the following has the most solutions in the interval  $[0, 2\pi]$ ?

- (A)  $\cos(100x) = 0$                       (B)  $\sin(100x) = 0$                       (C)  $\cos(101x) = 0$   
 (D)  $\cos(200x) = 1$                       (E)  $\sin(200x) = 1$

17. Let  $A$  be the point  $(1, 3)$ . Let  $B$  be the point  $(5, 11)$ .  $\triangle ABC$  is a right triangle with right angle at  $C$ . What is the largest that the area of  $\triangle ABC$  could be?

- (A)  $10\sqrt{3}$                       (B)  $20\sqrt{3}$                       (C) 40  
 (D) 80                      (E) none of these

18. Suppose that the highest point on the graph of  $y = f(x)$  is  $(3, 1)$  and that the lowest point is  $(5, -3)$ . What is the highest point on the graph of  $y = 3(f(2x - 1))^2 + 6$ ?

- (A)  $(3, 33)$                       (B)  $(2, 9)$                       (C)  $(5, 9)$   
 (D)  $(9, 33)$                       (E) None of these

19. A biased coin has a probability  $p$  of coming up heads, where  $p$  is a rational number that's less than  $\frac{1}{2}$ . If the probability of two consecutive flips of the coin being identical is  $\frac{145}{288}$ , what's the probability of getting two heads in a row?

- (A)  $\frac{5}{24}$                       (B)  $\frac{1}{144}$                       (C)  $\frac{71}{288}$   
 (D)  $\frac{121}{576}$                       (E) None of these

20. Which of the following is closest to  $(1 - 10^{-100})^{10^{100}}$ ?
- (A) 0                      (B) 0.2                      (C) 0.3                      (D) 0.7                      (E) 1
21. Two vertical walls and a horizontal floor meet in the corner of a room. An ant, starting from the corner, walks 1 meter across the floor along a line at a  $45^\circ$  angle to both walls. Another ant, starting from the point 1 meter above the corner, walks 2 meters up the wall along a line at a  $45^\circ$  degree angle from horizontal. Find the distance between the two ants.
- (A)  $\sqrt{3 + 4\sqrt{2}}$    (B)  $\sqrt{2 + 4\sqrt{3}}$    (C)  $\sqrt{4 + 2\sqrt{2}}$    (D)  $\sqrt{4 + 3\sqrt{2}}$    (E)  $\sqrt{3 + 3\sqrt{2}}$
22. If  $n$  is a positive integer, what is the largest solution to  $\cos(nx) = \frac{1}{2}$  for  $0 \leq x \leq 2\pi$ ?
- (A)  $\pi + \frac{2n\pi}{3}$                       (B)  $\frac{5\pi}{3n}$                       (C)  $2\pi - \frac{n\pi}{3}$   
(D)  $2\pi - \frac{\pi}{3n}$                       (E) None of these
23. Suppose that we choose a random value  $\theta$  in the interval  $[0, 2\pi]$ . Which of the following is most likely to happen?
- (A)  $-\cos(\theta) > 0 > -\sin(\theta)$   
(B)  $\sin(\theta) > \cos(\theta) > -\cos(\theta)$   
(C)  $\cos(\theta) > \sin(\theta) > -\sin(\theta)$   
(D)  $\sin(\theta) > \cos(\theta) > -\sin(\theta) > -\cos(\theta)$   
(E)  $\sin(\theta) > \cos(\theta) > 0$
24. Suppose that we rotate the  $xy$ -plane  $45^\circ$  counterclockwise around the origin and then rotate  $45^\circ$  clockwise about the point  $(1, 1)$ . Which of the following describes the overall motion that has been performed?
- (A) A translation in the direction of the  $y$ -axis (positive or negative)  
(B) A translation in some direction other than that of the  $y$ -axis  
(C) A rotation by  $45^\circ$  (clockwise or counterclockwise) around some point  
(D) A rotation by some amount other than  $45^\circ$  around some point  
(E) None of the these
25. If  $x$  is an integer, which of the following must be a divisor of  $x^{320} - x^{170} - x^{151} + x$ ?
- (A) 4                      (B) 13                      (C) 17  
(D) 31                      (E) None of the these

## 2022 Answers / Level 3 Test

- |      |       |       |
|------|-------|-------|
| 1. E | 7. D  | 13. C |
| 2. B | 8. C  | 14. B |
| 3. D | 9. E  | 15. C |
| 4. D | 10. A | 16. C |
| 5. E | 11. A | 17. E |
| 6. A | 12. A | 18. A |

19. D

20. C

21. C

22. D

23. A

24. B

25. D