College of Charleston
Math Meet 2003
Written Test – Level 2

1. If the cosine of an acute angle is $1/3$, what is the cosine of an angle half that size?

   (A) $\dfrac{1}{6}$ (B) $\dfrac{2}{3}$ (C) $\sqrt{\dfrac{1}{6}}$

   (D) $\sqrt{\dfrac{2}{3}}$ (E) $\sqrt{\dfrac{1}{6}+\dfrac{2}{3}}$

2. Let A be the set of even integers, B be the set of integers whose squares are less than 10, and $C = \{1, 2, 4\}$. How many elements are in the set

   $$(A \cap B) \cup C?$$

   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3. When $x^{13} - 2x^7 + 1$ is divided by $x + 1$, what is the remainder?

   (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

4. For what values of the variable c does the quadratic polynomial

   $$p(x) = 3x^2 - 2x + c$$

   have *two real* roots?

   (A) $0 < c < \dfrac{2}{3}$ (B) $0 \le c \le 13$ (C) $c \ge 6$

   (D) $c < \dfrac{1}{3}$ (E) $c > -\dfrac{1}{6}$

5. The sum of the solutions to the equation $2x^2 - 6x - 3 = 0$ is

   (A) $\dfrac{9}{4}$ (B) $\dfrac{3}{2}$ (C) 3 (D) 6 (E) 0

6. Simplify the following expression:

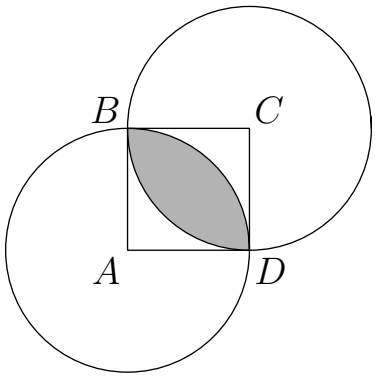
   $$\frac{\dfrac{a}{b} + \dfrac{b/a}{a-b}}{a}.$$

   (A) $\dfrac{a - b + b^2}{a^2 b(a-b)}$ (B) $\dfrac{a^3 - a^2 b + b^2}{ab(a-b)}$

   (C) $\dfrac{a^2 - ab + b^2}{b(a-b)}$ (D) $\dfrac{a-b}{ab}$

   (E) none of the above

7. The square $ABCD$ in the figure has sides of length 1. Both of the circles pass through the points B and D, but one is centered at A and the other at C. What is the area of the intersection of the two circles (the shaded region in the figure)?



(A) $\frac{\pi}{4}$ (B) $1-\frac{\pi}{4}$ (C) $\frac{\pi}{2}-1$

(D) $\frac{3\pi}{4}-1$ (E) $\frac{1}{3}$

8. Let A_n be the average of all whole numbers between 1 and 101 which are multiples of n. (So, for example, A_2 is the average of the numbers $\{2,4,6,\ldots,100\}$.) Which of these is the *largest*?

(A) A_2 (B) A_3 (C) A_4 (D) A_5 (E) A_6

9. The product of the complex numbers $3+8i$ and $-2i$ is equal to

(A) $-6-16i$ (B) $3+8i$ (C) $-16-6i$

(D) $3+6i$ (E) none of these

10. Which of these, when graphed as equations in polar coordinates, will result in a straight line?

I. $r\sin(\theta)-\cos^2(2\theta)=\sin^2(2\theta+2\pi)$
II. $r\left(\cos(\theta)-\sin(\theta)\right)=0$
III. $r\cos(\theta)=2r\sin(\theta)+5$

(A) I only (B) II only (C) III only

(D) II and III only (E) I, II and III

11. Find the units digit of $1!+2!+3!+4!+\cdots+1000!$

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

12. A man ordered a box of dishes from a website. When they arrived, he found only two of the dishes were undamaged. In fact, 2/3 of the dishes were cracked, 1/2 of them were chipped, and 1/4 of them were both cracked and chipped! Let n be the number of dishes he ordered. Which of the following is true of n?

(A) n is a multiple of 6 (B) n is a multiple of 5

(C) the sum of the digits of n is 8 (D) n is a prime number

(E) n is a perfect square

13. Alan sketched the graph of the function $f(x)$ and his friend Ellen sketched the graph of $g(x) = 1 - f(x-2)$. Which of these would describe what you could do to Alan's graph to turn it into Ellen's? (Note: These answers describe the steps in order from first step to last step, and the word "flip" refers to reflecting the graph vertically across the x-axis.)

(A) shift up 1, flip, then shift right 2

(B) flip, shift up 1, then shift left 2

(C) shift up 1, flip, then shift left 2

(D) shift down 1, flip, then shift right 2

(E) shift down 1, flip, then shift left 2

14. Express the number

$$0.\overline{21} = 0.21212121\ldots$$

as a rational number a/b in lowest terms. What is $a + b$?

(A) 20 (B) 40 (C) 60 (D) 80 (E) 189

15. If you know that $\sin(a_1)\sin(a_2) \approx .25000$ and $\cos(a_1)\cos(a_2) \approx .62361$, then what can you say about $\cos(a_1 + a_2)$?

(A) It is negative.

(B) It is approximately $.87$.

(C) It is more than $\cos(a_1 - a_2)$.

(D) It is approximately $.37$.

(E) We cannot say anything about $\cos(a_1 + a_2)$ from the given information.

16. Which of the following sequences is *not* geometric?

(A) $27, 9, 3, 1, \ldots$ (B) $5, -15, 45, -135, \ldots$

(C) $x, x^3, x^5, x^7, \ldots$ (D) $1, 4, 9, 16, \ldots$

(E) $1, 0.2, 0.04, 0.008, \ldots$

17. Which of these is a valid description of the function

$$f(x) = \frac{\sin(2x)}{2\sin(x)}?$$

(A) It has the same value as the function $\cos(x)$ at any x the domain of f.

(B) It is a constant function.

(C) It takes only positive values.

(D) This is just another way to write the function $\sin^2(x)$.

(E) It gives the ratio of two sides of any triangle with one angle equal to x.

18. **This is a corrected version of this question. The one which actually appeared on the 2003 test contained an error.**
In a given forest, suppose there are more trees than there are leaves on any *single* tree. Which of the following must be true?

I. There exist two trees with a leaf in common.
II. There exist at least two trees which have the same number of leaves.
III. There exists a tree with no leaves.

(A) I only (B) II only (C) III only
(D) either II or III (E) either I or III

19. The function $\dfrac{x+1}{x^3+1}$ can be written as the sum of an even function and an odd function. Find the even function.

(A) $\dfrac{x^4-1}{x^3+1}$ (B) $\dfrac{x^4+1}{x^6+1}$ (C) $\dfrac{x^4-1}{x^6+1}$
(D) $\dfrac{x^4+1}{x^6-1}$ (E) $\dfrac{x^4-1}{x^6-1}$

20. Which of the following facts are true for all positive numbers x, y and h?

I. $x^{(y^h)} = (x^y)^h$

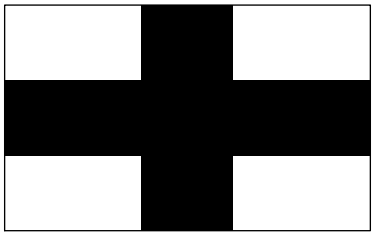
II. $\dfrac{\sin(x+h)-\sin(x)}{h} = 1$

III. $f(g(x)) = x$ if $f(t) = \sqrt{t}$ and $g(t) = t^2$

IV. $p(x+y) = p(x)+p(y)$ for any polynomial p.

(A) I and III (B) II and IV only (C) III only
(D) IV only (E) none are true

21. A flag is to be made with a black cross (all four arms the same thickness) centered on a 3 meter by 4 meter rectangle as shown in the figure. How wide should the arms of the cross be so that exactly half of the flag's area is white?



(A) $\dfrac{1}{2}$ m (B) $\dfrac{2}{3}$ m
(C) $\dfrac{4}{5}$ m (D) $\dfrac{8}{7}$ m
(E) none of the above

22. You have a hat with a white ball in it. You also have a paper bag with one white and one black marble in it. First, you randomly select one of the balls out of the bag (without looking at it) and place it in the hat. Then you randomly select one of the two balls out of the hat. If that ball is white, what is the probability that the ball still in the hat is also white?

(A) 0 (B) $1/3$ (C) $1/2$ (D) $2/3$ (E) 1

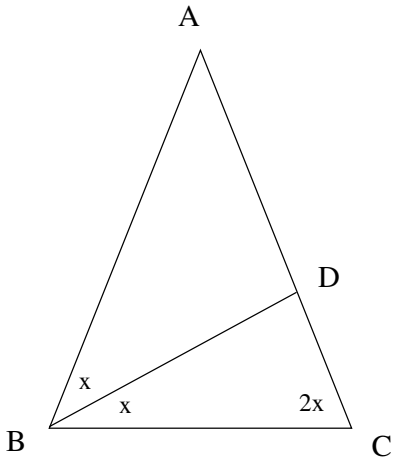
23. If you roll a pair of fair 8-sided dice (marked with the numbers 1 through 8 on each die), what is the most likely value for the sum of the two numbers rolled?

(A) 2 (B) 3 (C) 8 (D) 9 (E) 16

24. The 2003 inhabitants of an island are divided in two groups: the "truth tellers", who always tell the truth, and the "liars", who always lie. Each person is either a singer or a soccer player or a fisherman. We ask each inhabitant the following three questions: 1) Are you a singer? 2) Are you a soccer player? 3) Are you a fisherman? 1000 people answer "yes" to the first question, 700 people answer "yes" to the second question, 500 people answer "yes" to the third question. How many "liars" are there on the island?

(A) 105 (B) 183 (C) 197 (D) 319 (E) 732

25. Isosceles triangle ABC has the property that, if D is a point on AC such that BD bisects base angle ABC, then triangles ABC and BCD are similar. If BC has length 1, then AB has length



(A) $\sqrt{2}$ (B) $\dfrac{1+\sqrt{5}}{2}$

(C) 1 (D) 2

(E) none of the above

Answers

1. d
2. d
3. e
4. d
5. c
6. e
7. c
8. d
9. e
10. e
11. b
12. a
13. d
14. b
15. d
16. d
17. a
18. d
19. e
20. c
21. e
22. d
23. d
24. c
25. b