College of Charleston
Math Meet 2003
Written Test – Level 3

1. Suppose that f is an even, periodic function with period 2, and that $f(x) = x$ for all x in the interval $[0, 1]$. Find $f(3.14)$.

   (A) 3.14 (B) -3.14 (C) $-.14$ (D) $.14$ (E) $.86$

2. A cyclist is biking from his home to his job. Halfway there, he realizes that he has so far kept an average speed of 15 mile/hour. He decides to accelerate in order to achieve an average speed of 30 mile/hour for his entire trip. At what average speed must he cover the second half of his trip?

   (A) 40 mile/hour.

   (B) 45 mile/hour.

   (C) 60 mile/hour.

   (D) The answer depends on the total distance from home to work.

   (E) Regardless of the total distance, he now cannot average 30 mile/hour.

3. You extend each of the faces of a cube to an infinite plane. In how many regions do such planes subdivide the space which contains the cube (including the interior of the cube)?

   (A) 9 (B) 16 (C) 24 (D) 27 (E) 32

4. Determine the values of a and b so that the function

$$f(x) = \frac{x^3}{ax + b},$$

   has a local extremum at the point $\left(\frac{3}{4}, \frac{27}{32}\right)$.

   (A) $a = -2, b = 1$ (B) $a = 2, b = -1$

   (C) $a = -2, b = -1$ (D) $a = 2, b = 1$

   (E) None of the above

5. Find the union of all intervals having the form $\left[1 + \frac{1}{n}, 6 - \frac{2}{n}\right]$ where n is a positive integer.

   (A) $[1, 6]$ (B) $(1, 6)$ (C) $[2, 4]$ (D) $[\frac{3}{2}, 5]$ (E) $(1, 5)$

6. Compute the following limit:

$$\lim_{x \to 1} \frac{\sin^2(x^3 + x^2 + x - 3)}{1 - \cos(x^2 - 4x + 3)}.$$

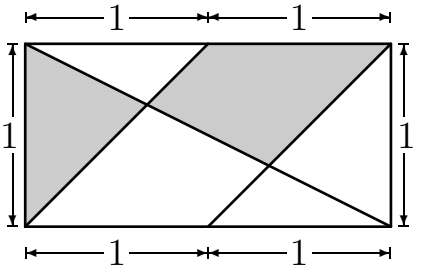
   (A) 10 (B) $\frac{3}{5}$ (C) 16 (D) $\frac{12}{7}$ (E) 18

7. The 2003 inhabitants of an island are divided in two groups: the "truth tellers", who always tell the truth, and the "liars", who always lie. Each person is either a singer or a soccer player or a fisherman. We ask each inhabitant the following three questions: 1) Are you a singer? 2) Are you a soccer player? 3) Are you a fisherman? 1000 people answer "yes" to the first question, 700 people answer "yes" to the second question, 500 people answer "yes" to the third question. How many "liars" are there on the island?

(A) 105 (B) 183 (C) 197 (D) 319 (E) 732

8. What is the area of the shaded region?

(A) $\frac{5}{6}$ (B) $\frac{4}{5}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) 1



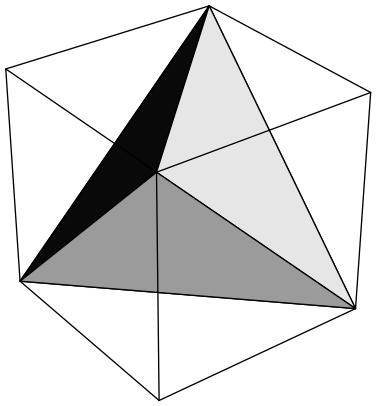
9. A recent poll of lowcountry students revealed the following: 8 liked pluff mud but not oysters, 11 liked boiled peanuts but not oysters, 9 liked boiled peanuts but not pluff mud, 4 liked oysters but not pluff mud, and 7 liked pluff mud but not boiled peanuts. How many students liked oysters but not boiled peanuts?

(A) 5 (B) 6

(C) 8 (D) 9

(E) Not enough information to tell

10. The shaded solid in the picture is a tetrahedron inscribed in a cube with side length one. As shown, two of the tetrahedron's vertices are the corners at the top of the cube in the front and the back and the other two vertices are the corners at the bottom of the cube at left and right. Pick the answer that is closest to the volume of this tetrahedron.



(A) $.1$ (B) $.2$ (C) $.3$ (D) $.4$ (E) $.5$

11. A committee of six people (Alan, Bert, Cathy, Dmitri, Eduardo, and Fulvio) must select its three officers: a chairperson, a secretary and a treasurer. How many different slates are possible in which Dmitri is either chairperson or treasurer and Eduardo is not an officer?

(A) 42 (B) 24 (C) 12 (D) 36

12. Five Hatfields and eight McCoys are to sit at a round table. After Pappy McCoy takes his seat, how many ways can the other 12 people be seated so that no Hatfield is next to another Hatfield?

(A) $\frac{7!5!}{2!}$ (B) $\frac{7!5!}{3!}$ (C) $\frac{5!8!}{3!}$ (D) $\frac{7!8!}{3!}$ (E) $\frac{8!8!}{3!}$

13. Nestor flips a fair coin eight times. What is the probability that the resulting sequence of heads and tails looks the same when viewed from either the beginning or from the end? (For example, HHHTTHHH is one such sequence.)

(A) $\frac{1}{32}$ (B) $\frac{1}{16}$ (C) $\frac{3}{16}$ (D) $\frac{3}{64}$ (E) $\frac{3}{32}$

14. Evanium is a radioactive substance which decays exponentially with time. In 2 hours, a sample of Evanium will decay to 90% of its original mass. How many hours will it take for a sample to decay to 10% of its original mass?

(A) 18 (B) $\frac{2}{1-\log 9}$ (C) $\frac{\ln 10}{\ln 10 - \ln 9}$

(D) $2\ln\left(\frac{1}{9}\right)$ (E) $-\log_{9/10} 10$

15. Find the number of real solutions to the equation

$$x^5 + x^4 + x^3 + x^2 + x + 1 = 0.$$

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

16. What is the derivative of the function

$$f(x) = x^2 + 2^x + e^x + e^2?$$

(A) $f'(x) = 2x + x2^{x-1} + e^x$

(B) $f'(x) = 2x + \ln(x)2^{x-1} + e^x + e^2$

(C) $f'(x) = 2x + 2^x + e^x$

(D) $f'(x) = 2x + 2^x + 2^x + e^2$

(E) $f'(x) = 2x + \ln(2)2^x + e^x$

17. If the graph of the polar equation $r = 1 + 3\sin\theta$ is rotated $\pi/2$ radians counterclockwise, find an equation for the resulting graph.

(A) $r = 1 + 3\sin(\theta + \pi/2)$ (B) $r = 1 + 3\cos(\pi/2 - \theta)$

(C) $r = 1 - 3\sin(\theta - \pi/2)$ (D) $r = 1 - 3\cos\theta$

(E) $r = 1 + 3\cos\theta$

18. You are standing at the point $(-4, 0)$ on the shore of a pond occupying the region $\{(x, y) : y > (x + 1)(x + 4)\}$. A friend is standing at the point $(4, 0)$ on the shore of a second pond occupying the region $\{(x, y) : y < (1 - x)(x - 4)\}$. You wish to walk to your friend along the shortest possible route, and you don't want to get your feet wet. Which of the following points will you cross on your path?

(A) $(2, 2)$ (B) $(-1/2, 7/4)$

(C) $(1, 0)$ (D) $(0, 1)$

(E) $(\sqrt{2}, 5\sqrt{2} - 6)$

19. Let A be the set of real numbers x for which $x^5 + x - 1 = 0$. Find the true statement.

(A) A is the empty set.

(B) A consists of exactly one point between $x = 0$ and $x = 1$.

(C) A consists of exactly one point between $x = -1$ and $x = 0$.

(D) A consists of exactly two points between $x = -1$ and $x = 1$.

(E) A consists of exactly three points.

20. If $f(x) = x^2$, find $\lim_{h \to 0} \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$.

(A) 1 (B) 2

(C) x (D) $2x$

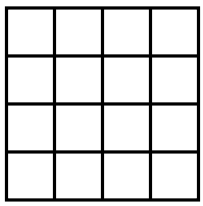
(E) The limit does not exist.

21. The function $\dfrac{x+1}{x^3+1}$ can be written as the sum of an even function and an odd function. Find the even function.

(A) $\dfrac{x^4-1}{x^3+1}$ (B) $\dfrac{x^4+1}{x^6+1}$ (C) $\dfrac{x^4-1}{x^6+1}$

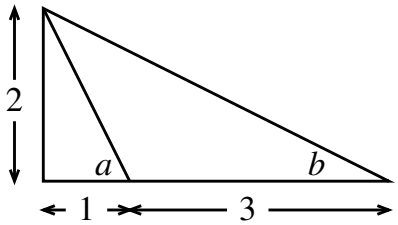
(D) $\dfrac{x^4+1}{x^6-1}$ (E) $\dfrac{x^4-1}{x^6-1}$

22. How many rectangles are in the accompanying figure?



(A) 30 (B) 50 (C) 60 (D) 81 (E) 100

23. Find the true statement about the angles a and b in the accompanying figure.



(A) $75^\circ < a + b \leq 80^\circ$ (B) $80^\circ < a + b \leq 85^\circ$

(C) $85^\circ < a + b \leq 90^\circ$ (D) $90^\circ < a + b \leq 95^\circ$

(E) None of the above.

24. Suppose $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$. Then x equals:

(A) $\dfrac{1 + \sqrt{13}}{2}$ (B) 2.25 (C) 2.5

(D) 3 (E) 3.2

25. Where does the tangent line to the graph of the function

$$f(x) = x^2 - 3x + 1$$

at the point $(0, 1)$ hit the x-axis?

(A) at $x = -1$ (B) at $x = 3$ (C) at $x = -\dfrac{1}{3}$

(D) at $x = \dfrac{1}{3}$ (E) it doesn't

Answers

1. e
2. e
3. d
4. b
5. b
6. e
7. c
8. a
9. a
10. c
11. b
12. d
13. b
14. b
15. e
16. e
17. d
18. a
19. b
20. b
21. e
22. e
23. c
24. d
25. d